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Analog Active Filters: State of the Art

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Review article

Analog Active Filters: State of the Art

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Abstract

In this tutorial paper, the state of the art of analog filter design is reviewed. The various design methodologies for Active RC filter design using operational amplifiers (opamps), Operational Transconductance amplifier (OTA)-C based filters, switched-capacitor (SC) technique , current-mode devices such as current conveyors, Current-feedback amplifiers are presented. The various design considerations and analysis techniques are highlighted. In adition, some of the recent appoaches for active filter design using transistors are also considered for completeness.

Keywords: Active RC filters, Analog Filters, SC Filters, Switched-Capacitor Technique, OTA-C Filters, Current-mode Filters, Sensitivity

I. INTRODUCTION

The subject of Active Filters has been extensively investigated in the past six decades. Several textbooks and collection of papers have been published [1]-[19]. With the advent of Digital Signal Processing (DSP), the Analog front-end used for band limiting the input signal to meet the requirements of sampling theorem, needs an anti-aliasing filter. The resulting output of the anti-aliasing filter is fed to the A/D converter. After the DSP operations such as amplification, automatic gain control (AGC), filtering, convolution, parameter extraction as needed in speech or video coding, encryption, protection against error correction etc. carried out by the DSP on the output samples of the A/D converter, the resulting digital information can be stored or transmitted. At the receiver, after performing the reverse operations performed at the transmitter, the digital information corresponding to the encoded analog signal needs to be converted to analog form. This will yield a stair case type of waveform also known as sampled and held signal. This signal needs to be smoothed to remove the high frequency components, which are multiples of the sampling frequency. The smoothing operation also needs a smoothing filter to yield a base-band analog signal. The complete information flow is sketched in Figure1.

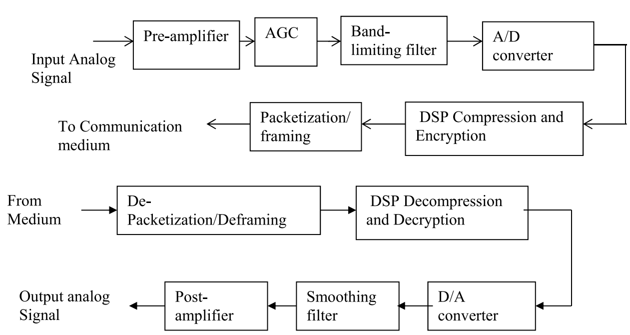

Figure 1. A typical Mixed-Signal System

The art of analog filter design has gone through several stages and it appears that it has gone back to the stage in 1950s albeit in a different setting. In this review and tutorial paper, we review the various approaches described in literature over the past few decades. In Section II, we present the vacuum tube and early transistor based filter design era. In Section III, the bipolar op-amp (Operational amplifier or OA) based filter design techniques are surveyed. In section IV, the CMOS op-amp based filter design techniques especially switched-capacitor and early continuous-time deigns are covered. In section V, we

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describe the filter design methodologies developed using OTAs and some current-mode devices are considered. In section VI, we present a few latest trends in active filter design in VLSI form.

II. TRANSISTOR BASED FILTER DESIGN

First-order passive RC low-pass and high-pass filters are shown in Figure 2 (a) and (b). They can realize negative real poles only. The realized transfer functions are

$$\frac{V_o}{V_i} = \frac{1}{1+sCR} \text{ and } \frac{V_o}{V_i} = \frac{sCR}{1+sCR} \qquad (1)$$

respectively. The 3dB cut off frequency is $\omega_o = \frac{1}{CR}$.

Before the advent of op-amp, vacuum tubes initially and later bipolar transistors were used to realize filters. The passive networks were used together with one inverting or non –inverting gain block realized using transistors. Sallen and Key [20] have published a catalog of second-order low-pass, high-pass and band-pass filters (see Table I for all possible second-order transfer functions). The most popular among these is configuration of Figure 3 (a) which uses an emitter follower stage to realize second order low-pass and high-pass filters. Note that non-inverting amplifier of gain >1 also has been suggested by Sallen and Key [20] but this needs more number of transistors.

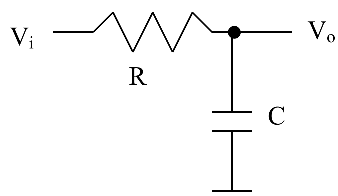
In another design, Twin-T (or parallel-T) RC null network [21] has been used in the negative feedback path of a common emitter configuration as shown in Figure 3(b) [22]. Notable applications were in rejecting certain frequency by using in common collector configuration [23]. Even techniques or controlling the sharpness of notch (today known as high pole-Q) were envisaged using partial positive feedback (Figure 3 (c)) using R_E and R'_E. Sharpening

Table I. Second-order transfer functions†

Type	Transfer function	Dc gain	Gain at ∞	Remarks
Low-pass	$\frac{\omega_p^2}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	1	0	*
High-pass	$\frac{s^2}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	0	1	*
Band-pass	$\frac{s\frac{\omega_p}{Q_p}}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	0	0	Centre frequency gain = 1
Notch	$\frac{s^2+\omega_z^2}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	1, $\omega_z = \omega_p$	1	Useful for rejection of a certain frequency
Low-pass notch	$\frac{s^2+\omega_z^2}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	$\frac{\omega_z^2}{\omega_p^2}$, $\omega_z > \omega_p$	1	*Used in Elliptic filters
High-pass notch	$\frac{s^2+\omega_z^2}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	$\frac{\omega_z^2}{\omega_p^2}$, $\omega_z < \omega_p$	1	*Used in Elliptic filters
All-pass	$\frac{s^2-s\frac{\omega_p}{Q_p}+\omega_p^2}{s^2+s\frac{\omega_p}{Q_p}+\omega_p^2}$	1	1	Used for delay equalizer

* Peaking can occur based on Q_p value.
†Gain of the biquad is not considered

the frequency response using tapering also has been well investigated. Several other null networks [24]-[26] have also been studied in place of using Twin-T null network. Oscillators using Twin-T network in the grounded base configuration also have been described [27] (see Figure 3 (d)) and extended to realize band-pass filters [28] (as shown in dotted lines in Figure 3 (d)). Alternative techniques using network synthesis concepts using negative impedance converters also have been described. Note however that the analysis of the circuits using the transistor equivalent circuit shows that the finite input resistance of the bipolar transistor and finite load resistances used at the collector and emitter terminals affects the performance. The design is slightly difficult due to the lack of easy tunability which led to the circuit design using opamps in seventies once operational amplifiers were available as ICs such as µA741, µA709 etc.



(a)

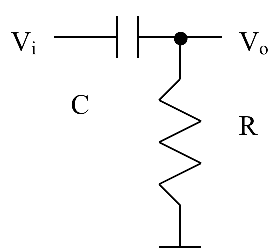


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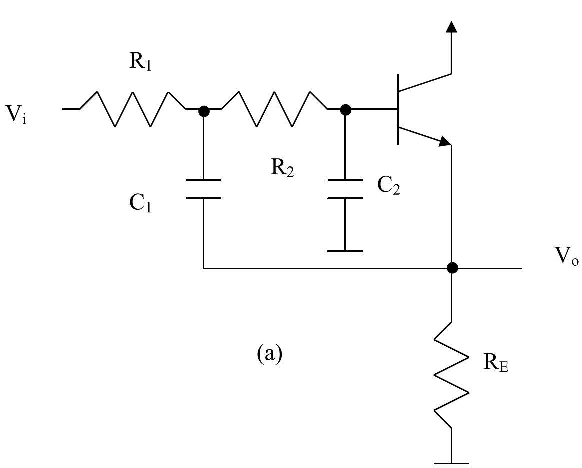
Figure 2 (a) and (b). First-order low-pass and high-pass filters



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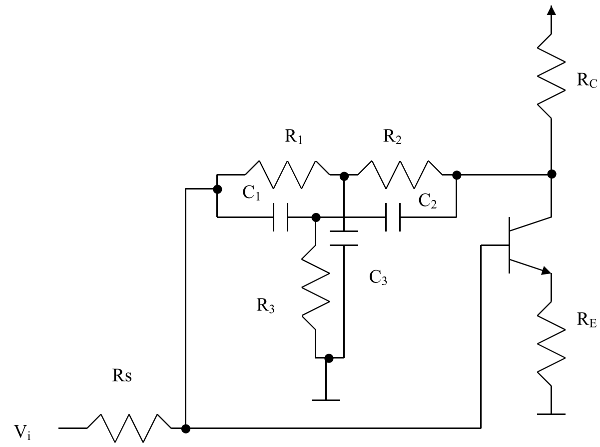

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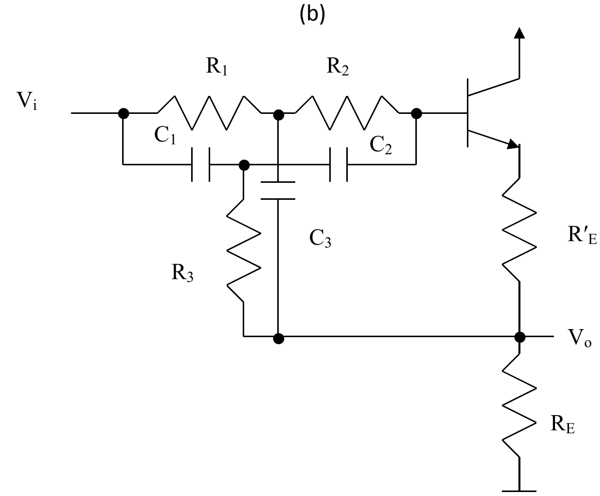

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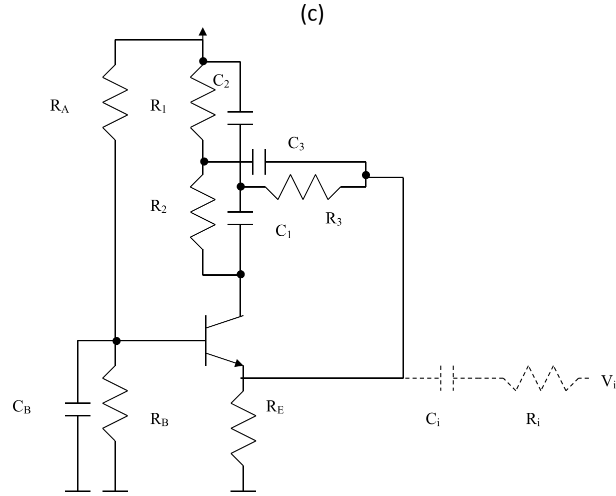


(d)

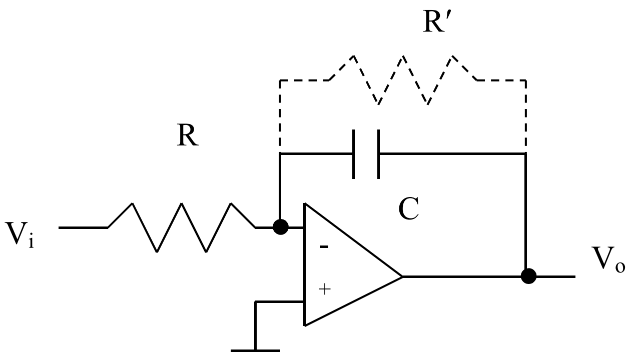
Figure 3. (a) Sallen-key low-pass filter using Emitter follower (b) Twin-T based notch filter using emitter follower (c) Twin –T based band-pass filter (d) Emm's oscillator using common base amplifier (and modification to realize a filter shown in dotted lines).

III. OP-AMP BASED FILTER DESIGN

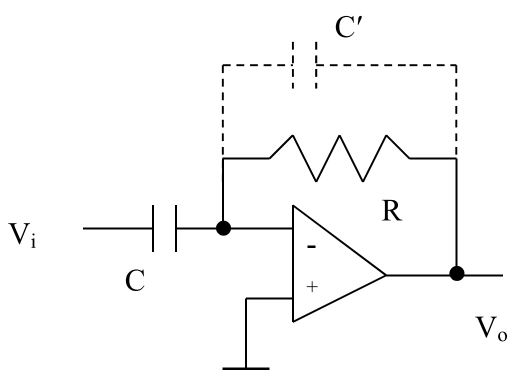
There is a flood of literature on Active *RC* filter design using op-amps. The basic building blocks are integrator and differentiator shown in Figure 4 (a) and (b). The transfer functions are respectively given as

$$\frac{V_o}{V_i} = -\frac{1}{sCR} \text{ and } \frac{V_o}{V_i} = -sCR \qquad (2)$$

By shunting a resistor R_1 across the integrating capacitor in Figure 4 (a) or by shunting additional capacitor C′ across the feedback resistor in Figure 4 (b) (as shown in dotted lines), a lossy integrator (first-order low-pass filter) or a first-order high pas filter can be obtained. A first-order all-pass filter can be obtained using the circuit of Figure 4 (c). We next consider the realization of second-order filters.



(a)



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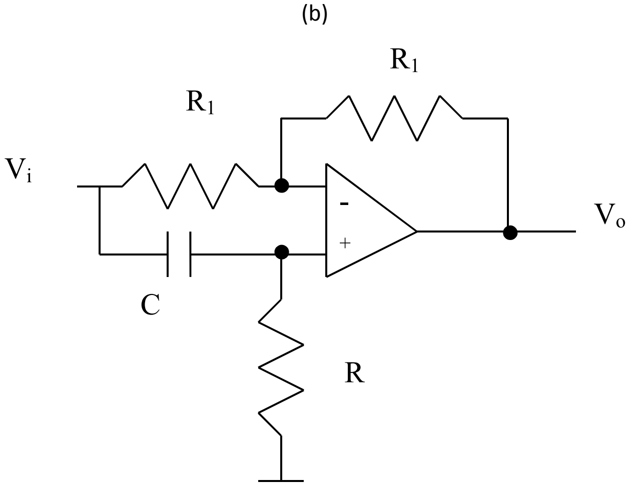
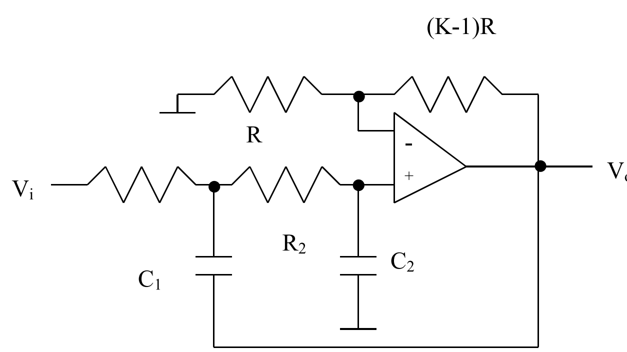


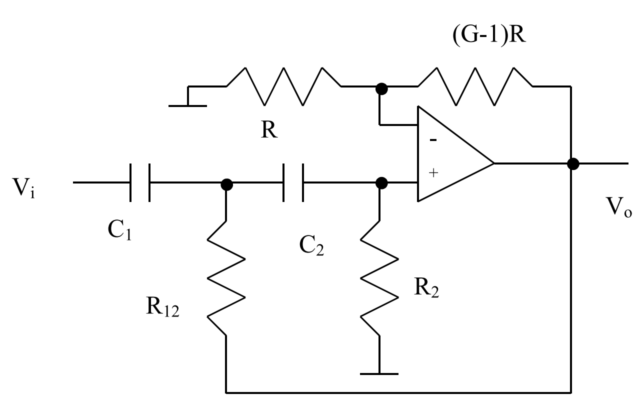
(c)

Figure 4. (a) Active RC integrator (b) Active RC differentiator (c) First-order all-pass filter

The well-known Sallen and Key filter [20] of Figure 5 (a) is the simplest among these for realizing second-order low-pass filters. The advantage over transistor-based designs is that the trade-off between component spread in values (ratio of resistors R_1 and R_2 values, and ratio of capacitors C_1 and C_2) with the gain of the non-inverting amplifier gain *K*. The concept of *RC*: *CR* transformation (replacing resistors with capacitors and capacitors with resistors of frequency determining components only; not those deciding the gain of the amplifier) yield second-order high-pass filters (see Figure 5 (b). The transfer function of the Sallen and Key low-pass filer of Figure 5(a) is given as



(a)



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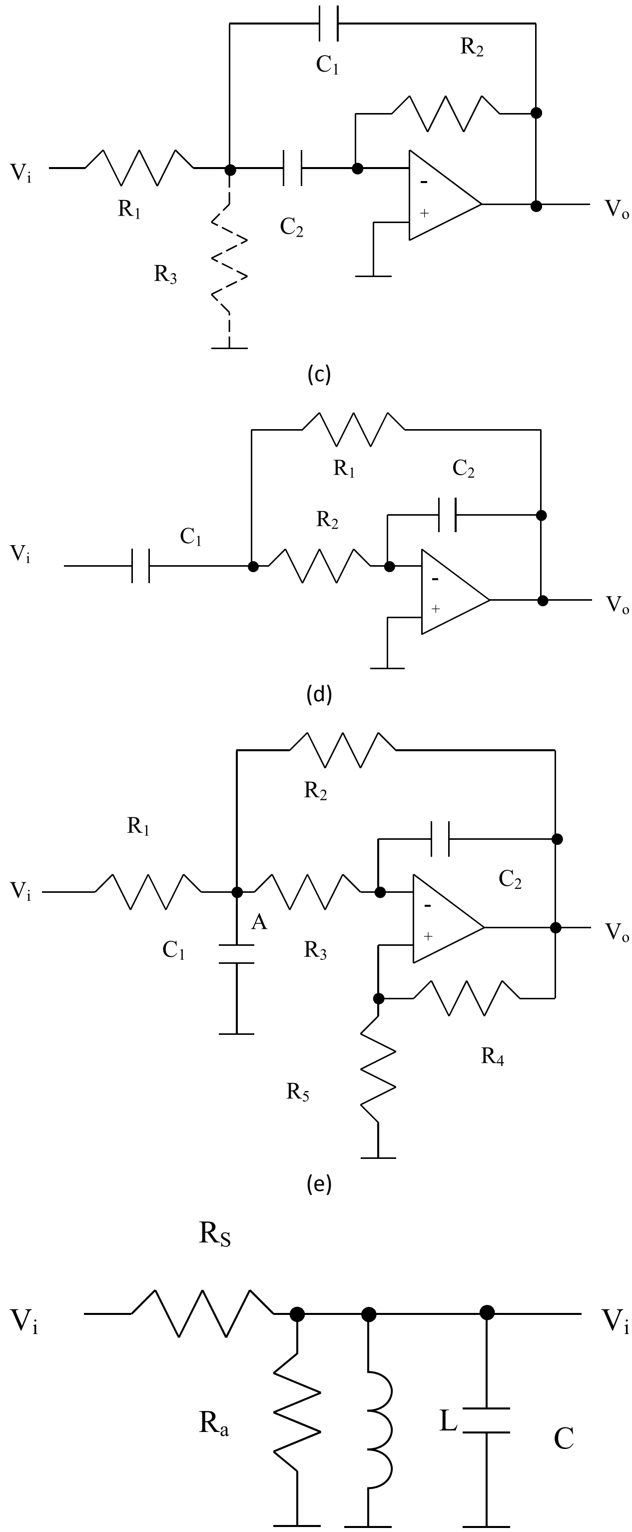


Figure 5. (a) Sallen and Key Low pass filter (b) Sallen and Key High-pass filter (c), (d) Multiple feedback band pass filters and (e) Multiple feedback low-pass filter (f) RLC band-pass filter

$$\frac{V_o}{V_i} = \frac{K}{s^2 C_1 C_2 R_1 R_2 + s\ (C_2 R_2 + C_1 R_1 (1-K) + C_2 R_1) + 1} \quad (3a)$$

When $C_1 = C_2$ and $R_1 = R_2$, (3a) simplifies as

$$\frac{V_o}{V_i} = \frac{K}{s^2 C^2 R^2 + s\ CR(3-K) + 1} \quad (3b)$$

The pole-frequency and pole-Q are given as $\omega_p = \frac{1}{CR}$ and $Q_p = \frac{1}{3-K}$.

We introduce the concept of sensitivity next. Due to fabrication tolerances, the values of passive components and parameters like gain etc. of active components vary from the nominal (desired) values. The effect of these variations to the realized pole frequency and pole-Q can be analyzed using the sensitivity values. The sensitivity of a parameter *x* to a component *y* is defined as

$$S_y^x = \frac{dx}{dy} \times \frac{y}{x} = \frac{dx/x}{dy/y} \quad (4)$$

This computes the percentage change in the parameter *x* due to percentage change in component value *y*. As an illustration, from (3c), we can write $S_K^{Qp} = \frac{K}{3-K} = 3Q_p - 1$. It can thus be seen that for a filter with Q_p = 10, the percentage change in Q_p for 1% change in K is 29%, not acceptable good design. The sensitivities shall be computed for the original formulas i.e., from pole-frequency and pole-Q expressions from (3a) rather than from (3b). Thus, for the circuit of Figure 5(a), we need to find sensitivities of ω_p and Q_p to R_1, R_2, C_1 and C_2 and K. As an illustration, the sensitivities of ω_p to R_1, R_2, C_1, and C_2 are all -0.5. This means that a 1% increase in R_1 or R_2 or C_1 or C_2 decreases the pole frequency by 0.5%.

A second-order band-pass filter configuration and the circuit obtained by *RC*:*CR* transformation from it are shown in Figure 5(c) and (d). Note that the center (pole) frequency can be controlled independent of bandwidth by incorporating an additional resistor R_3 [29] as shown dotted lines in Figure 5(c). The transfer function of this circuit is given as

$$\frac{V_o}{V_i} = \frac{+s\ C_2 R_2}{s^2 C_1 C_2 R_1 R_2 + s\ R_1 (C_1 + C_2) + 1 + \frac{R_1}{R_3}} \quad (5)$$

A low-pass filter is presented in Figure 5(e) whose DC gain is decided by resistors R_1 and R_2. *RC*:*CR* transformation yields a second-order high-pass filter. Note that at the node *A* in Figure 5(e), a second-order band-pass transfer function is obtained when

$R_5 = 0$ and $R_4 = \infty$ (i.e., no positive feedback is used). This suggests by analogy with the band-pass filter circuit using *R*, *L*, and *C* shown in Figure 5(f) that a lossy 'simulated' grounded inductance is realized by op-amp, C_2, R_2, and R_3. Ford and Girling [30] suggested this. Note that all the circuits of Figure 5 require large spread in component values of the order of Q_p^2 where Q_p is the pole-*Q*.

The realization of notch or all-pass transfer function can be realized using the band-pass filters of Figure 5(c) and (d) by feeding the attenuated input signal to the non-inverting terminal of the op-amp as shown in Figure 6(a) and (b). Note that α can be used realize notch transfer function or all-pass transfer function. The transfer function of the circuit of Figure 6(a) is given by

$$\frac{V_o}{V_i} = \frac{\alpha s^2 C_1 C_2 R_1 R_2 + s\ (\alpha R_1(C_1+C_2) - C_2 R_2(1-\alpha)\ + \alpha}{s^2 C_1 C_2 R_1 R_2 + s\ R_1(C_1+C_2)+1} \quad (6a)$$

Note that Notch and all-pass transfer functions can be obtained when

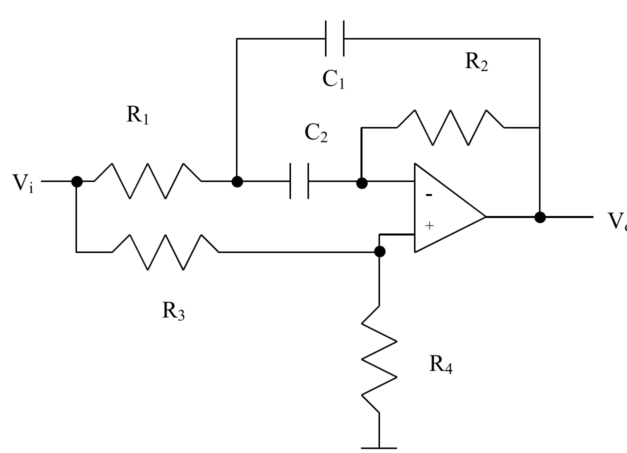
$$\alpha = \frac{C_2 R_2}{R_1(C_1+C_2)+C_2 R_2} \text{ and } \alpha = \frac{C_2 R_2}{2R_1(C_1+C_2)+C_2 R_2} \quad (6b)$$

respectively. All the circuits described above can realize only one type of transfer function (low-pass, band-pass, or high-pass) types. Hence, there was interest to have only one building block denoted as *biquad* (meaning that the transfer function is quadratic in numerator and quadratic in denominator) which can be configured to realize various types of second-order transfer functions using a single op-amp but a few resistors and capacitors. A topology due to Friend [31], [32] shown in Figure 6 (c) can realize all second-order transfer functions except low-pass type. The low-pass transfer function can be realized using another circuit (see Figure 5(e)) which uses positive feedback as shown in the Multiple-feedback low-pass filter of Figure 5(e). The design starts assuming C_1, C_2 values and proceeds to determine all other component values. An all-pass filter can be realized using the circuit of Fig. 6 (d). Friend's biquad has been realized as STAR (Standard Tantalum Active Resonator) in thin film integrated circuits.

Initial active filter designs were for very low-frequency applications in telephony. However, it was found that active filters designed for pole frequencies of <10 KHz and moderate pole-Q were showing tendency to be unstable. Several authors have diagnosed the reason to be the finite bandwidth of the op-amp. Since bipolar op-amp was designed as three-pole system with a dominant pole at very low frequency and two poles at high frequency greater than the unity gain bandwidth, the frequency dependent open loop gain of opamp can be modeled as integrator with a transfer function given by

$$A(\omega) = -\frac{B}{s+\omega_a} \cong -\frac{B}{s} \quad (7)$$

where $B = A_o \times \omega_a$ and A_o is the open-loop dc gain. Using this model to analyze second-order filters using single op-amp has shown that the excess phase shift of the op-amp due to the integrator behaviour in (7) is magnified by the pole-Q of the circuit and if the Barkhausen criterion for instability is met, the system will oscillate. Closed form expressions for the deviation in pole-frequency and pole-Q could be derived so that the designer can know the effect of non-ideal op-amp. The pole movement trajectory as a function of bandwidth also could be plotted to show the instability. As an illustration, the transfer function of the Sallen and Key [20] filter of Figure 5 (a) taking into account the op-amp model of (7) can be derived as



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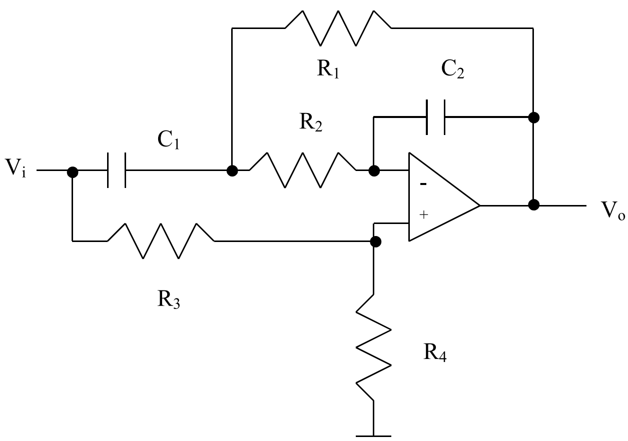

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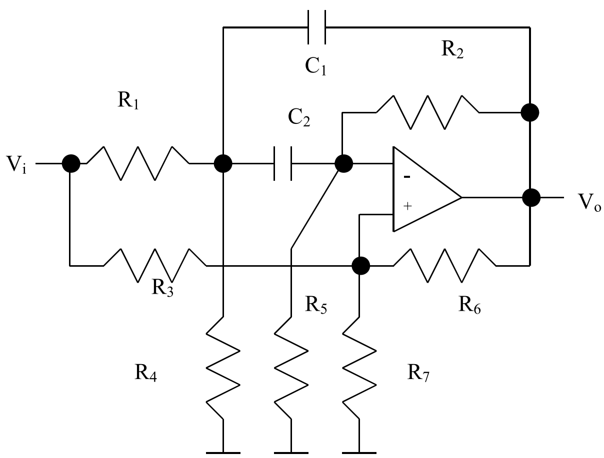

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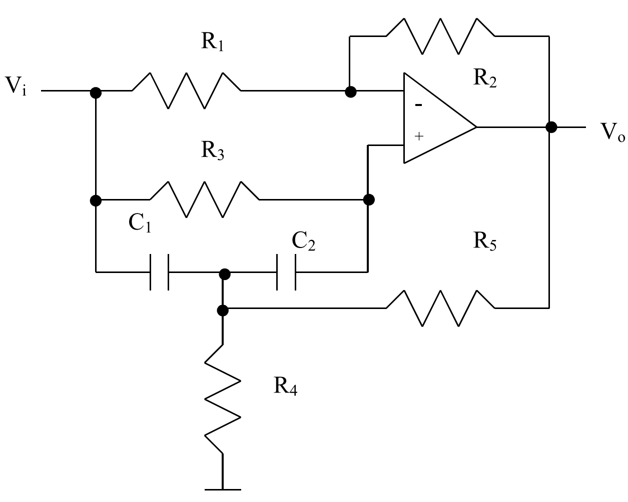


(d)

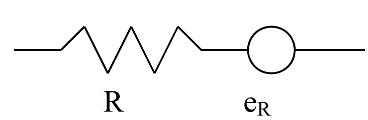
Figure 6. (a), (b) Notch filters based on band-pass filters of Figure 5. (c) and (d), (c) Friend's biquad and (d) Steffen all-pass filter.

$$\frac{V_o}{V_i}=\frac{K\omega_o^2\left(1+\frac{Ks}{B}\right)}{s^3\frac{K}{B}+s^2\left(1+\frac{3K\omega_o}{B}\right)+s\left((3-K)\omega_o+\frac{K\omega_o^2}{B}\right)+\omega_o^2} \quad (8)$$

This is a third-order transfer function. The shifted pole frequency and shifted pole-Q can be derived by using the Akerberg-Mossberg approximation [33] for the high-Q case by substituting $s^3=-s\,\omega_o^2$ which makes the third-order transfer function a second-order transfer function. The effect of non-ideal op-amp (finite bandwidth) and trade-off between the passive and the active sensitivities for a SAB (single amplifier biquad) of Figure 6(c) are thoroughly investigated by Fleischer [34].

Interestingly, methodologies for analysis of output noise of second-order filters due to Johnson noise of resistors (see Figure 7 (a)) and input referred noise voltage and current of op-amp (see Figure 7 (b)) have been developed to yield close form solutions and expressions for total output noise of a biquadratic filter [35].

Since single op-amp based filters can realize only one transfer function at the output of a op-amp and do not have *orthogonal* (independent) tunability of pole-frequency, pole-Q and gain, second-order filters using two or more op-amps have been developed. Among the two op-amp structures, GIC based biquad [36] is most interesting. A GIC based bandpass filter is shown in Figure 8. Note that at node A, the input impedance is given by $Z_{in} = Z_1Z_3Z_5/ (Z_2Z_4)$. By choosing Z_2 or Z_4 as a capacitor and Z_1, Z_3, and Z_5 as resistors, Z_{in} becomes a grounded lossless inductance. The circuit can be generalized to realize a biquadratic transfer function by using feed-forward branches from input to the various internal nodes. An interesting device known as *Frequency Dependent Negative Resistance* (FDNR) [37] can be obtained by choosing Z_2 and Z_4 as capacitors i.e., Z_2=1/(sC_2) and Z_4=1/(sC_4) and Z_1, Z_3, Z_5 as resistors R_1, R_3, and R_5 respecively. The resulting input impedance is $Z_{in}= s^2C_2C_4R_1R_3R_5$. Substituting $s = j\omega$, we have $Z_{in} = -\omega^2C_2C_4R_1R_3R_5$ which is negative resistance and is frequency dependent due to the ω^2 term. This is useful in realizing high-order filters based on RLC filters that will be considered later.



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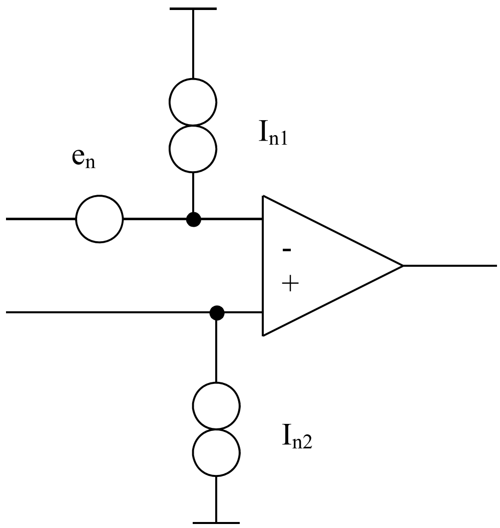


(b)

Figure 7. Noise model of a resistor (a) and op-amp (b)

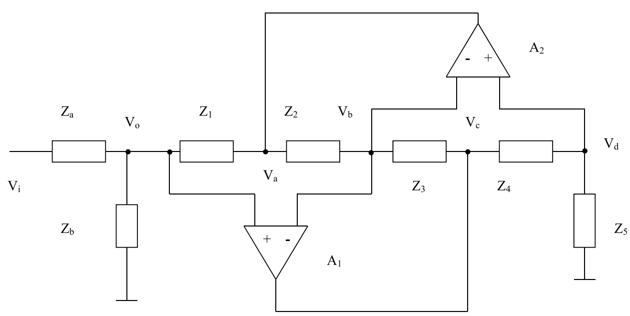


Figure 8. Active filter using GIC

Several three-amplifier active RC biquads have been described in literature. Most prominent among them is Kerwin-Huelsman-Newcomb (KHN) biquad [38] (see Figure 9(a)) which realizes LP, BP, and HP transfer functions at the three output terminals of three op-amps. It uses two lossless integrators in a negative feedback loop. The pole frequency can be controlled by R_5, C_1, R_6, R_3, R_4, and C_2 whereas the two resistors R1 and R2 decide the pole-Q. The gain cannot be controlled. It has been later found [39] that a notch transfer function is also available at *X* terminal. The transfer unction of the KHN biquad under the condition $R_3 = R_4$ is given by

$$\frac{V_o}{V_i} = \frac{\frac{2R_2}{R_1+R_2}}{s^2C^2R^2 + s\ CR\frac{2R_1}{R_1+R_2} + 1} \qquad (9)$$

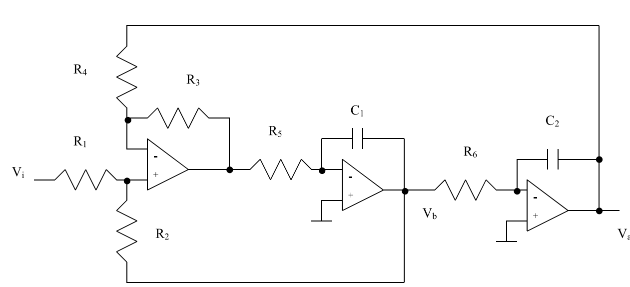
A variation of the KHN biquad (also known as state-variable biquad) is Tow-Thomas biquad [40], [41], which uses an inverting lossy integrator (formed by R_3, R_2, C_1) and non-inverting lossless integrator (formed by R_4, R_5, R_6, C_2) in a feedback loop (see Figure 9 (b)). This circuit realizes only low-pass and band-pass transfer functions. This circuit has the advantage of independent control of pole frequency, pole-*Q*, and gain using three separate controls. The spread of components is in the order of Q_p only. The transfer function of the Tow-Thomas biquad is given by

$$\frac{V_o}{V_i} = \frac{-\frac{R_3}{R_1}}{s^2\frac{C_1C_2R_4R_6R_3}{R_5} + s\frac{C_2R_4R_6R_3}{R_5R_2} + 1} \qquad (10)$$

Choosing $R_4 = R_5$, $R_6 = R_3 = R$, $C_1 = C_2 = C$, $R_2 = Q_pR_3$, (10) simplifies as

$$\frac{V_o}{V_i} = \frac{-\frac{R_3}{R_1}}{s^2C^2R^2 + s\frac{CR}{Q_p} + 1} \qquad (11)$$

Note that the DC gain is R_3/R_1, pole-Q controlled by R_2 and pole-frequency by C_1 and C_2. The effect of op-amp finite bandwidth is to generally decrease the pole frequency and increase the pole-Q sometimes leading to instability. Solutions for compensation known as *passive compensation* (using capacitors or resistors) [41] and *active compensation* (using additional op-amps) have been described [42].



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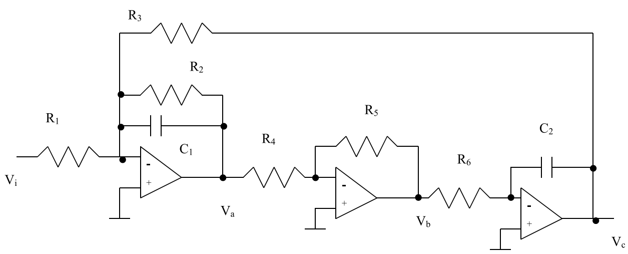


(b)

Figure 9. (a) KHN biquad (b) Tow-Thomas biquad

The problem of finite bandwidth of the op-amp causing deviations in performance was addressed in two different ways. On one technique, since op-amp was an integrator, it was used together with an external capacitor and resistors to realize a filter. These filters were known as *partially Active-R* filters. In fact, the well-known differentiator of Figure 4(b) behaves like a band-pass filter (see Figure 10(a)) and using an external resistance, it can be converted to a band-pass filter with variable bandwidth as well [43]. The explanation for this stems from the RLC model once again. The circuit shown in dotted lines in Figure 10 (a) simulates a lossy grounded inductance at terminal A. The transfer function of the circuit of Figure 10(a) can be obtained as

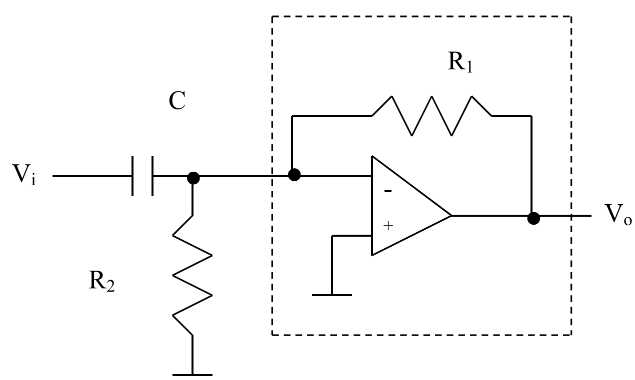
$$\frac{V_o}{V_i} = \frac{-sCR_1}{\frac{s^2 C R_1}{B} + \frac{s}{B}\left(1 + \frac{R_1}{R_2}\right) + 1} \tag{12}$$

Thus, the pole-frequency and pole-Q are dependent on the bandwidth of the op-amp and C, R_1 and R_2 values.

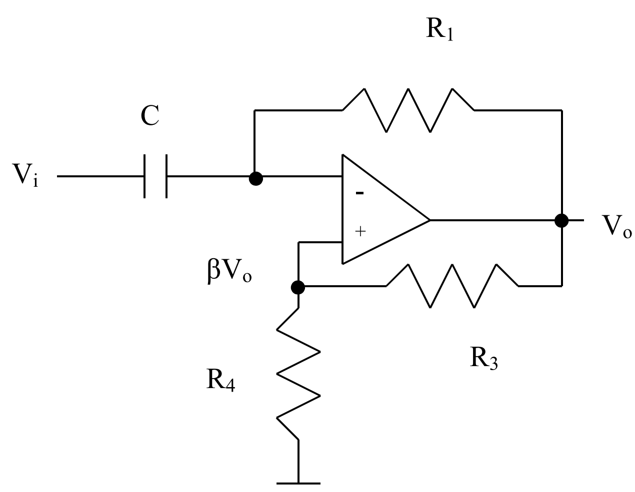
One very well known relaxation oscillator using op-amp shown in Figure 10(b) has been shown [44] to be a filter with excessive positive feedback and uses the amplifier pole to realize an oscillator. Another interesting partially active R filter due to Radhakrishna Rao and Srinivasan [45] is shown in Figure 10(c) from which a very simple low-pass/band-pass filter can be obtained as shown in Figure 10(d) [46].

Another interesting technique has considered removing the need for even one capacitor but instead using the finite bandwidths of two op-amps. These can be understood by looking at Tow-Thomas Biquad, which uses two Active RC integrators. This led to the concept of Active R filters (see Figure 10(e)) using only resistors and op-amps. These have the advantage that pole-frequency, pole-*Q*, and gain depend on ratio of resistors whereas the pole frequency depends on absolute value of the bandwidths. The transfer function of this circuit can be derived as

$$\frac{V_{o2}}{V_i} = \frac{\frac{1}{\beta}}{\frac{s^2}{\beta B_1 B_2}\left(1 + \frac{R_3}{R_5}\right) + \frac{s}{\beta B_1} + 1} \tag{13}$$



(a)



(b)

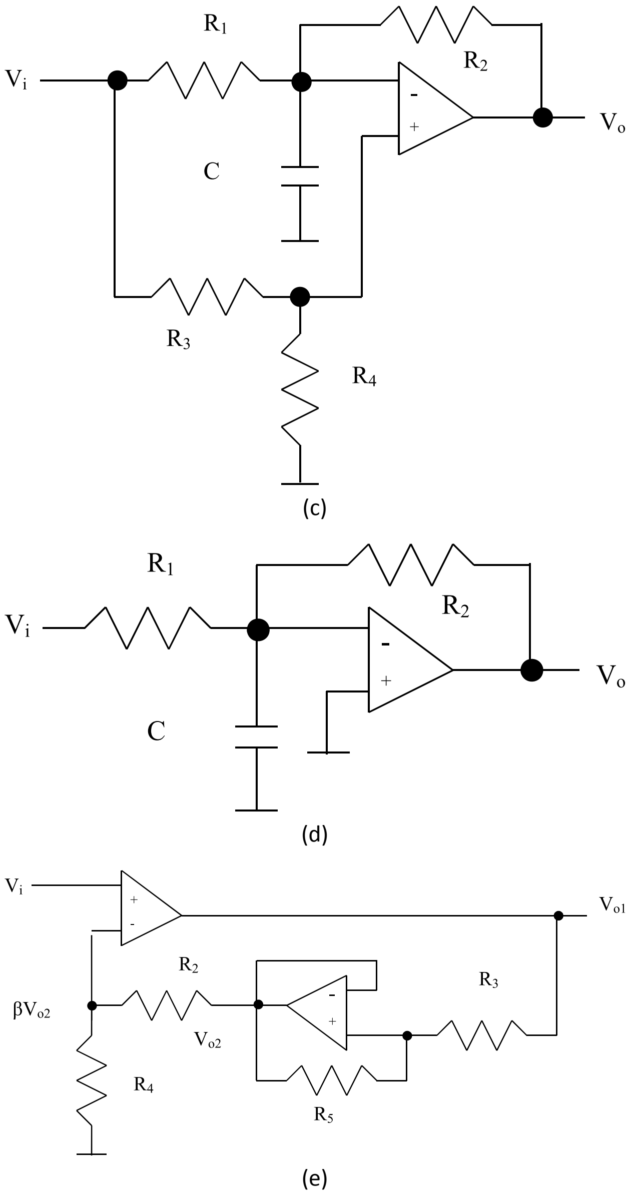


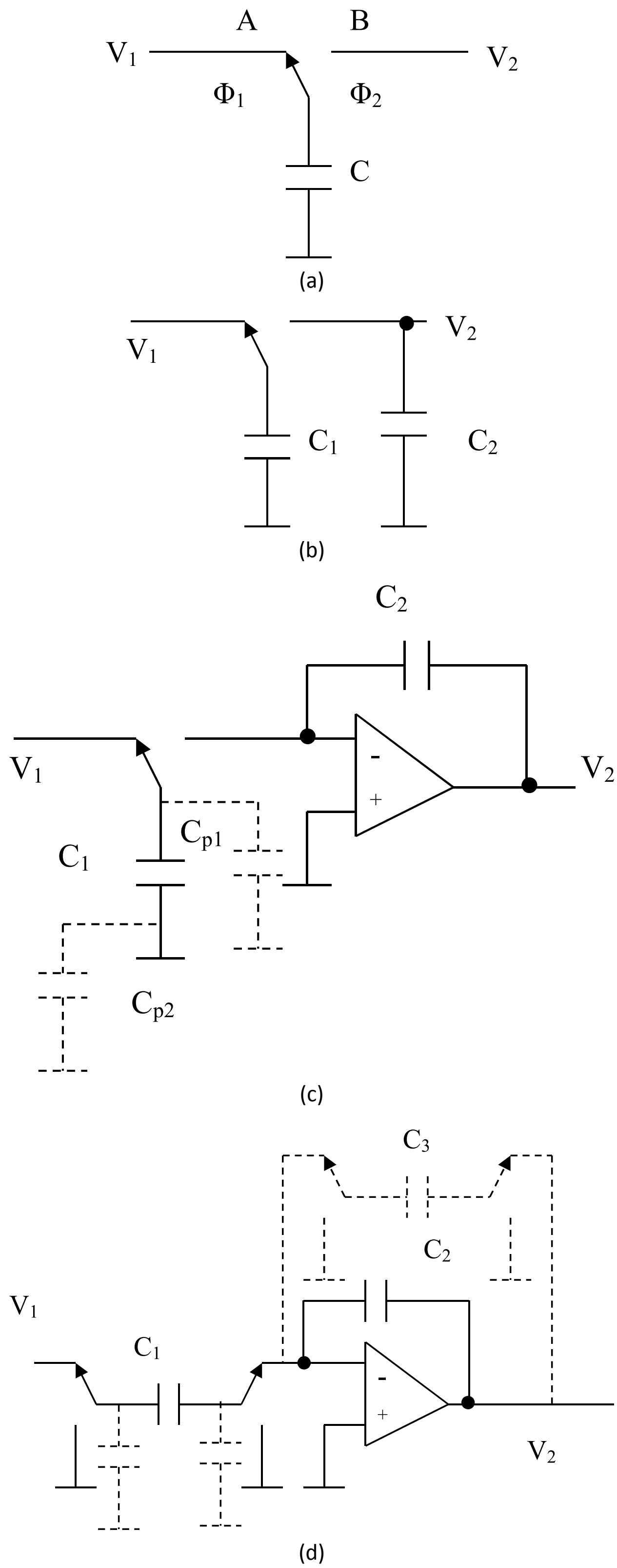
Figure 10. (a) A partially Active R bandpass filter (b) An oscillator using amplifier pole (c) Rao and Srinivasan Partially Active R filter (d) A simplified Low-pass/bandpass filter and (e) An Active R filter

Thus, the pole-frequency is dependent on bandwidths of both the op-amps, β, R_3 and R_5. Unfortunately, since the Op-amp bandwidth varies with temperature and power supply voltage, the performance (pole frequency) depends on the power supply voltage and temperature. Techniques such as Follow-the-master [47] have been suggested to maintain the pole frequency at the desired value. These need extra hardware.

Next generation Active filters were based on the Switched-capacitor technique [48], [49]. A resistor can be realized using two switches and a capacitor as shown in Figure 11 (a). A two-phase clock drives the switches. The capacitor is charged from one phase to V_1 and discharged to V_2 from another phase. Hence, effectively an incremental charge of $C_1(V_1\text{-}V_2)$ flows between the terminals A and B in a time interval T. The current through the equivalent resistor is defined as

$$i=\frac{\Delta Q}{T}=\frac{C(V_1-V_2)}{T}=\frac{(V_1-V_2)}{(T/C)} \tag{14}$$

Effectively, a resistance $R = T/C$ is realized between the terminals A and B. It is interesting to note that by substituting the SC realized in a RC first-order filter (see Figure 11(b)), we have the time constant $R_1C_2 = TC_2/C_1$ a value which is dependent on ratio of capacitors and a sampling clock of period T. Since T can be controlled using a stable clock, precise pole frequency of the filter can be obtained. The SC of Figure 11 (a) can be used to realize a lossless integrator as shown in Figure 11 (c). Note, however, that the capacitor C_1 will have parasitic capacitances C_{p1} and C_{p2} at both the top and bottom plates respectively. In the circuit of Figure 11(c), the bottom plate is grounded making this parasitic C_{p2} not affecting the performance. On the other hand, the parasitic at the top plate adds to the actual capacitance thereby changing the time constant. An ingenious solution to this problem is to realize the SC in a stray-insensitive manner using the circuit of Figure 11(d). Both the parasitic capacitances C_{p1} and C_{p2} do not switch between ground and source or ground and virtual ground making the circuit stray-insensitive. Interestingly, by interchanging the two switches as in Figure 11(d), we can realize inversion as well considering a non-inverting lossless integrator as shown in Figure11 (e).These two circuits are the most important building blocks for SC filters. Note that by connecting an SC resistor across the integrating capacitor, we obtain a lossy integrator as shown in dotted lines in Figure 11(c) and (d). A second-order filter can be built by connecting a lossy integrator and lossless integrator in a feedback loop one being inverting and the other non -inverting type. Such a Biquad is shown in Figure 11(f), which is due to Fleischer and Laker [50]. Here, the capacitors G,H,I and J realize the transmission zeroes whereas the poles are realized using the feedback loop comprising A,B,C,D,E and F. Either $E = 0$ or $F = 0$ can be used to realize the damping (i.e., pole-Q) and the desired output can be taken at either op-amp output terminal T or T'.



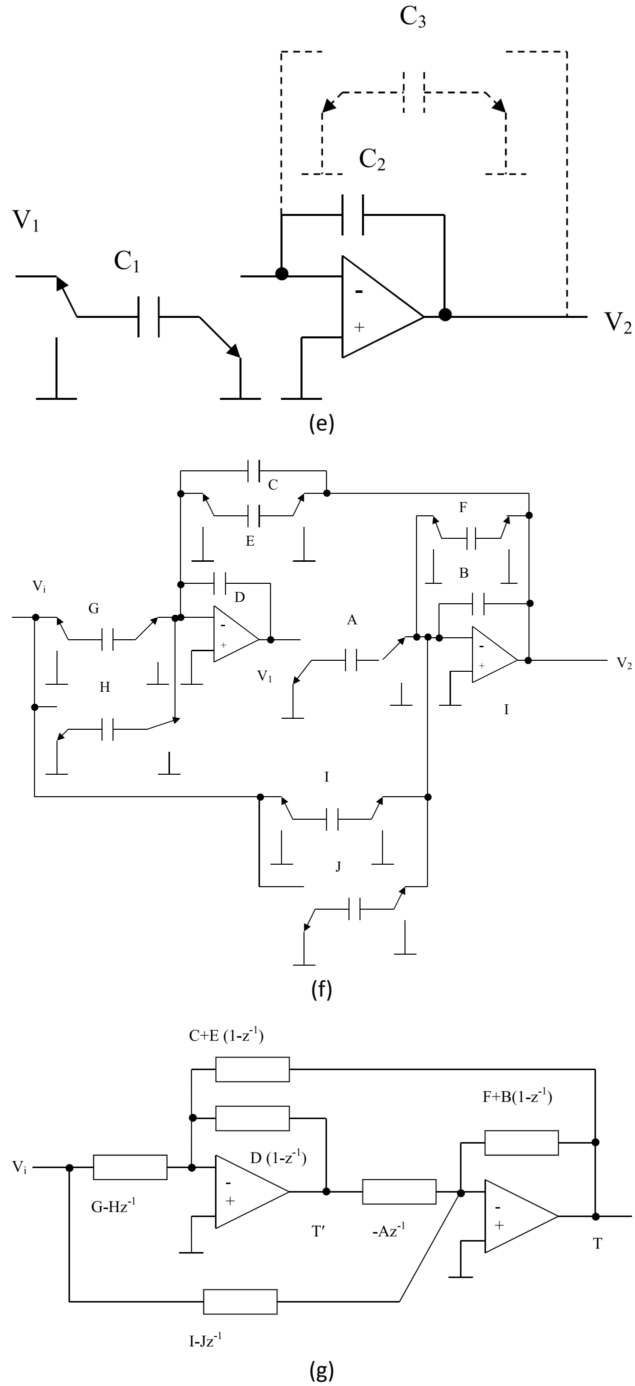


Figure 11. A resistor using SC, (b) first-order low –pass filter using (a), (c) a SC inverting integrator (d) inverting integrator (e) non-inverting integrator (f) Fleischer-Laker Biquad and (g) z-domain equivalent of (f).

The SC filters are the sampled-data circuits since at discrete time instants, the charge transfers occur between various capacitors. Hence, they are analyzed using DSP techniques. First obtaining z-domain equivalent circuit of the SC network can analyze SC filters by using incremental charge –

voltage (ΔQ (z) -V (z)) relationships similar to I(s) –V(s) relationships in continuous-time RLC circuits. This approach yields a much simpler circuit where all branches are capacitive (see Figure 11(f)). Next, we can write charge conservation equations similar to Kirchoff's current law at all nodes. These equations can be solved to obtain the *z*-domain transfer functions. As an illustration, the transfer functions at T and T′ outputs of the SC filter of Figure 11(e) are as follows:

$$T = \frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(F+B) + [A(C+E) - DF - 2DB)]z^{-1} + (DB - AE)z^{-2}} \quad (15a)$$

$$T' = \frac{\{I(C+E) - G(F+B)\} + [H(F+B) + BG - JC - E(I+J)]z^{-1} + (EJ - BH)z^{-2}}{D(F+B) + [A(C+E) - DF - 2DB)]z^{-1} + (DB - AE)z^{-2}} \quad (15b)$$

Thus, the body of digital filter design techniques can be used to arrive at digital transfer functions that can be matched with those of chosen SC filters to obtain the various capacitor ratios. After this step, several optimizations for minimum area, minimum ratio of capacitors, scaling or optimal dynamic range, and minimization of number of switches can be carried out to obtain the best SC filter.

Since the SC filters need op-amps, which shall be able to drive capacitive load only, the conventional op-amps using three stages to be able to drive resistive load are not needed. Hence, either two-stage compensated op-amps or two stage op-amps with no compensation for which the load capacitance can act as compensation capacitor are used. The SC filters have problems due to CMOS op-amp offset voltage, noise of op-amps, noise of MOS switches (and the resulting fold-over noise), clock feed-through of switches and finite gain of the CMOS op-amps (typically about 1000). Solutions are available for these. SC ladder filters are also possible in a similar way as leapfrog ladder Active RC filters. The SC filters are affected by other op-amp non-idealities like power supply rejection ratio etc. Hence, the Active Filters of next generation have reverted to continuous-time filters.

Using MOS FET as a resistor was the first idea that was well known in 1970's. However, in one class of CT filters, the non-linearity of the MOS (high-order distortion) has been shown to be eliminated by using fully differential topology [51] as shown in Figure 12. Note, however, that now the pole frequency depends on the absolute values of simulated resistances and capacitors. In order to bring the pole frequency to the desired value, *follow-the-Master* technique [47] needs to be used. We need to have a CT oscillator whose frequency will be brought to lock with an external high stability clock using a phase locked loop by controlling the gate voltage of the MOS transistors used to realize the resistors. This control voltage can be used to control the transistor gates in the actual filter that needs to be realized. Hence, the frequency control loop is an overhead in the case of CT filters. Nevertheless, only one such circuit is needed for a complex filtering system.

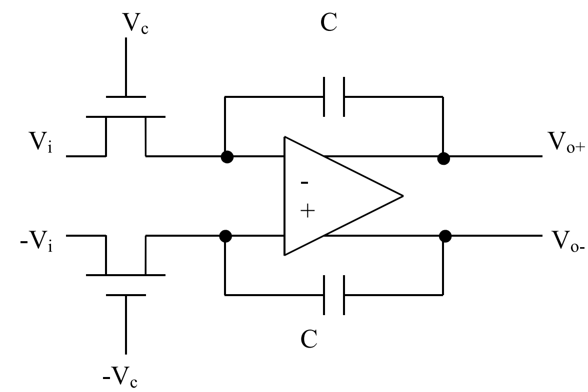


Figure 12. A Continuous-time filter

IV. OTA-C BASED FILTER DESIGN

The necessity to drive a small capacitive load of < 100pf has led to using open loop voltage to current converters or so called Transconductances or *Operational transconductance amplifiers* (OTA). These are represented by the symbol shown in Figure 13 (a). The OTA is described by the equation

$$I = g_m(V_1 - V_2) \quad (16)$$

It can perform non-inverting or inverting V-I conversion i.e., converting the differential input voltage to current. OTA can be used as a resistor using the arrangement of Figure 13 (b). Inverting and non-inverting lossless integrators can be easily realized as shown in Figure 13 (c) and inverting and non-inverting lossy integrators can be realized using g_{m2} as shown in dotted lines in Figure 13 (c). A Two-integrator loop can be easily realized as shown in Figure 13 (d). The transfer function of this circuit is given as

$$\frac{V_{o1}}{V_i} = \frac{s\,C_2 G_{m1}}{s^2 C_1 C_2 + s\; C_2 G_{m3} + G_{m4} G_{m2}} \tag{17}$$

The finite output resistance and capacitance of the OTA as shown in dotted lines in Figure 13 (a) which can be absorbed in the physical impedances connected at the output terminals in some cases affect the OTA-C filters [52].

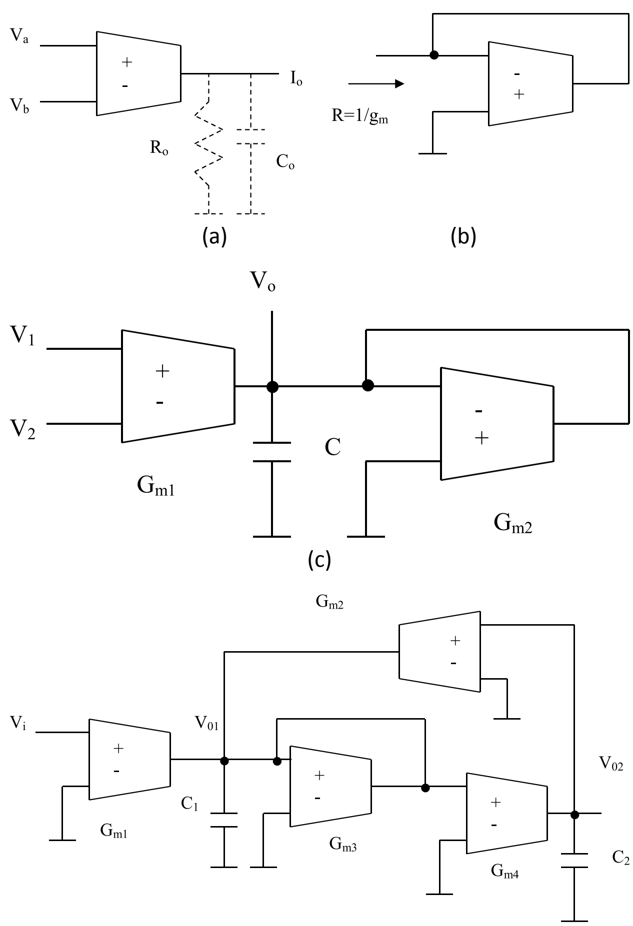


Figure 13. (a) OTA symbol (b) resistor simulation using OTA (c) A OTA-C integrator and (d) a OTA-C biquad

OTAs can be used together with capacitors to realize floating inductances as shown in Figure 14 (a) by the block shown within dotted lines between nodes A and B. A third-order elliptic filter derived from the prototype of Figure 14(b) is shown in Figure 14 (a). Hence, the high-order ladder filters can be easily realized. Note that g_{m1} and g_{m5} realize the source and the load resistances R_S and R_L in Figure 14(b).

Actually OTA-based filters are mixed–mode filters that mean that they can take current or voltage inputs and deliver voltage or current outputs. OTA-C filters also need pole-frequency and pole-Q control loops for bringing the transconductances to the desired values. They have the advantage that Q and gain are dependent on ratio of transconductances.

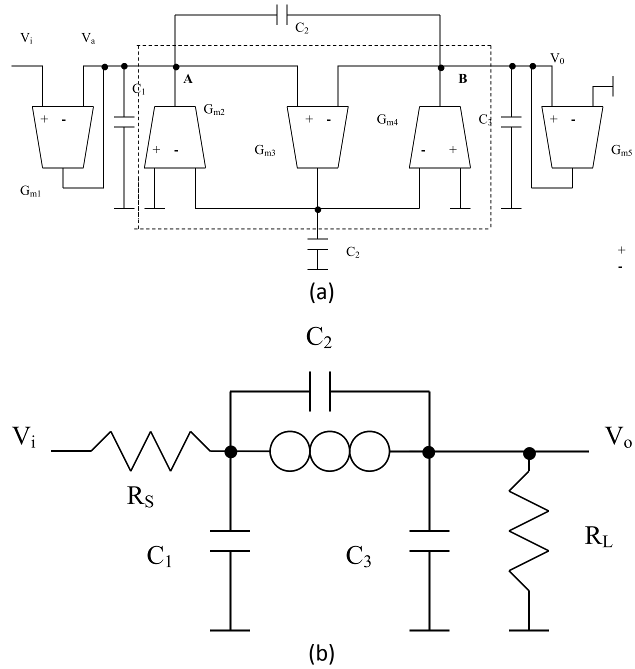


Figure14. A OTA –C third-order elliptic filter (a) based on component simulation of the prototype of (a)

V. CURRENT-MODE FILTERS

Interestingly, bipolar transistor is a current-mode device and Emm's oscillator [27] and band-pass filter using Twin-T network [28] of Figure 3 (d) are examples of first current-mode filters. In 1970's, however, a new device known as *second-generation current conveyor* (CCII) was introduced [53]. This is a three terminal device (see Figure 15 (a)) with $i_z = i_x$, $i_y = 0$ and $v_x = v_y$. The model is shown in Figure 15 (b). The terminal *x* is the current-input and has the input resistance R_x of about a few tens of Ohms. Such devices are available in market from Analog Devices (AD 844) [54]. The output resistance and capacitance at *z* terminal are two Meg Ohms and 4.5pF respectively. AD 844 also buffers the output voltage at *z* at *w* terminal as shown in dotted lines in Figure 15 (a). This device known as *current feedback amplifier* (CFOA) can also be used to build filters, which are Mixed-mode (current or voltage/input or output).

Simple lossy integrator and a biquad using CCII are shown in Figure 15 (c) and (d). The interesting point is that a simple inverting amplifier using CFOA exhibits constant bandwidth independent of gain unlike an op-amp in which case the bandwidth decreases with increase in gain: $B_{eff} = B/(G+1)$. Unfortunately, CFOA cannot be easily used to realize an integrator.

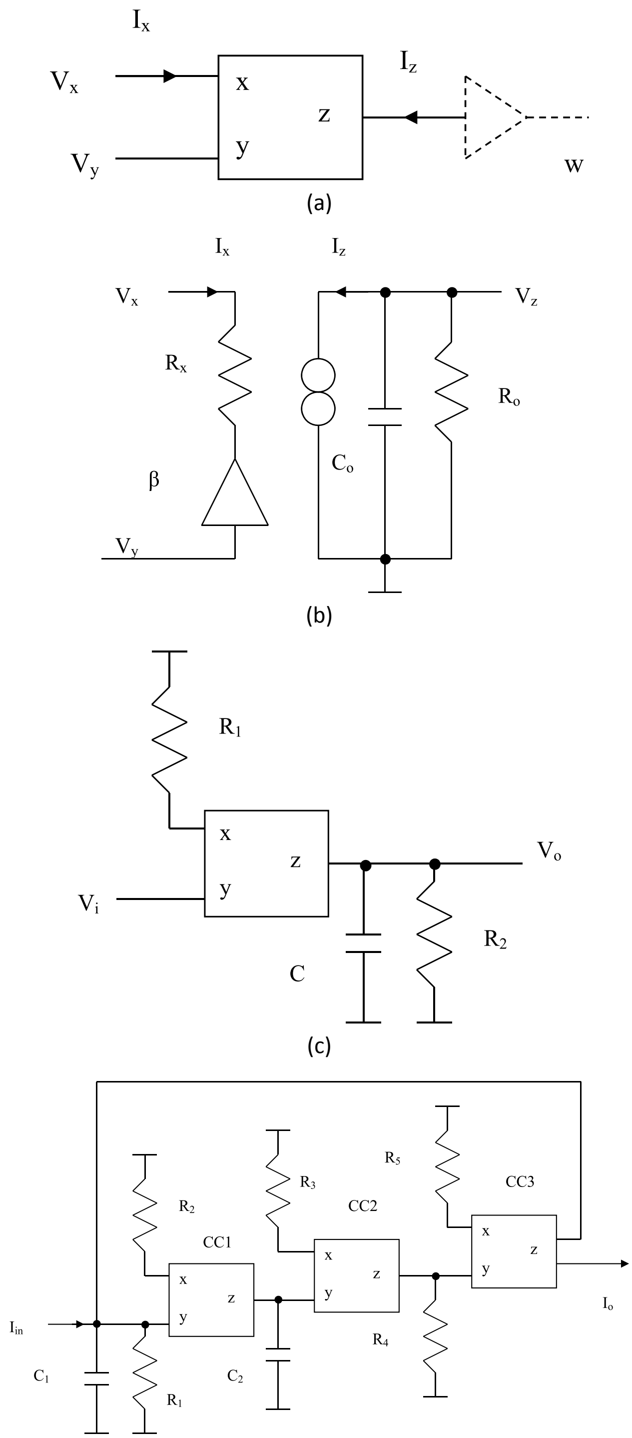


Figure 15 (a) a Current conveyor symbol (b) model of (a) (c) A CC based lossy integrator (VIVO) and (d) A CC based biquad (CICO)

It may be noted that contemporary Mixed-Signal system designers use one of the previously described techniques for realizing filters. However, recently, emphasis has been on low power and low area designs. Hence, the designers have preferred to not using the building blocks like OTAs, op-amps but prefer transistor- based circuits or circuits using just buffer amplifiers.

VI .RECENT APPROACHES

One recent design has used partially-active R filters since the fourth order low-pass filter needs only two op-amps, two capacitors and four resistors [55], [56]. More recent designs used for single transistor based active filters [57] only three components decide the filter characteristics excluding the biasing arrangement that may need resistors, current sources etc., (e.g., see Figure 16 (a) and (b)) which have used inductances also. Another notable design of beam forming filters [58] has used first-order all-pass filters using one transistor and one resistor and capacitor and other passive components for biasing (see Figure 16 (c)). The effect of virtual ground node not being at perfect ground potential due to the finite frequency dependent gain of the op-amp (see (7)) has been solved by using a negative resistance at the virtual ground terminal[59],[60]. This idea was described originally by Boutin [61] and explored in [62] (see Figure 16 (d)). More recently, the SC filters using buffers and multiplexing have been integrated (Figure 16 (d)) [63], [64] which again is a case of reinventing history of Bach's configuration [65] to meet the current requirements.

A single op-amp second-order filter for CT sigma-delta modulator using parallel-T RC network [66] is shown in Figure 16(e). This circuit realizes a transfer function with independent control of poles and zeros:

$$\frac{V_o}{V_i} = \frac{1}{(C_2C_3+C_2C_3)R_{in}} \cdot \frac{(C_1+C_2)s+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}{s^2+\frac{1}{(C_1+C_2)C_3R_1R_2}} \qquad (18a)$$

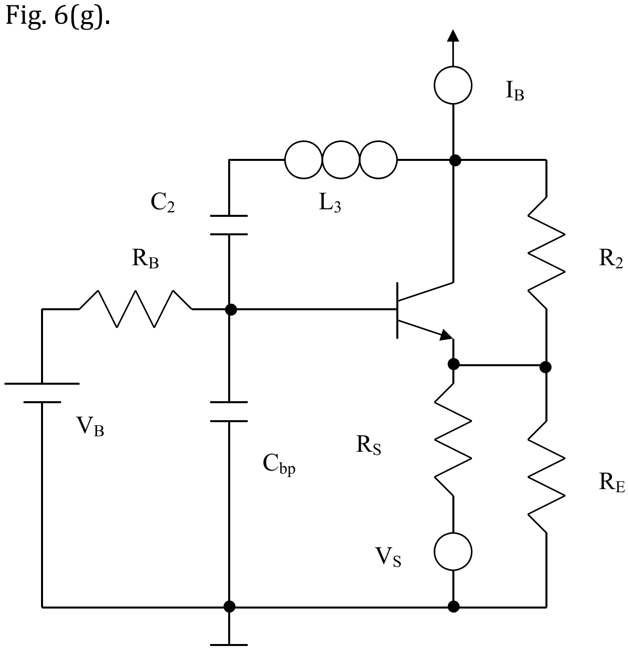
under the condition

$$C_2R_2R_3 = C_3(R_1R_2+R_2R_3+R_3R_1) \qquad (18b)$$

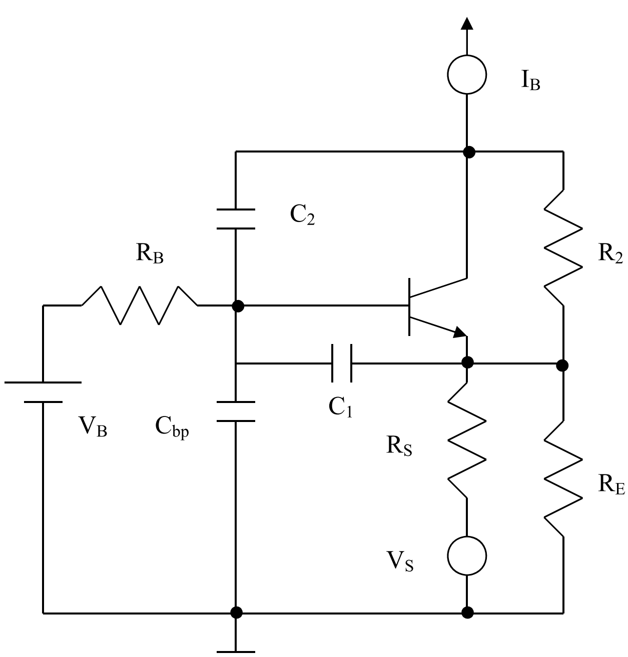
A super-source-follower based low-pass analog filter using only two capacitors and four transistors for realizing 33MHz cut-off frequency has been recently described [67]. The transfer function of this circuit shown in Fig. 6(f) is given as

$$\frac{V_o}{V_i} = \frac{A_o}{1 + s\frac{C_1}{g_{m1}} + s^2\frac{C_1 C_2}{g_{m1} g_{m2}}} \quad (19)$$

Grarakoui et al [68] have described a g_m-C cascadable all-pass true delay cell with variable delay using transistors and a programmable capacitor shown in Fig. 6(g).



(a)



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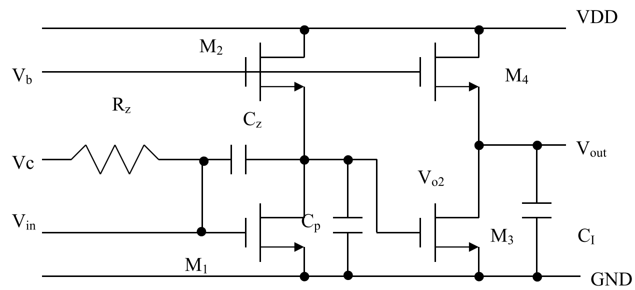
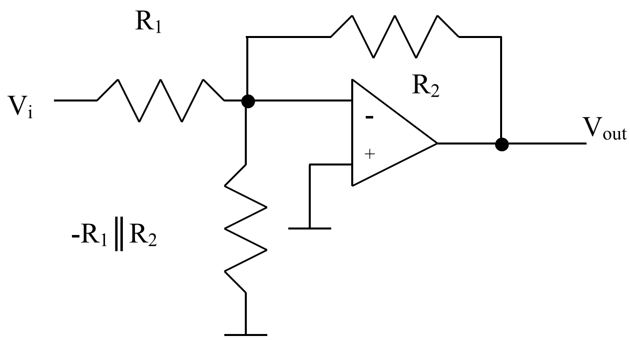
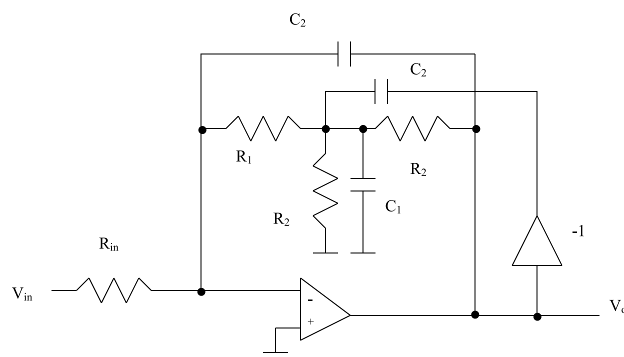


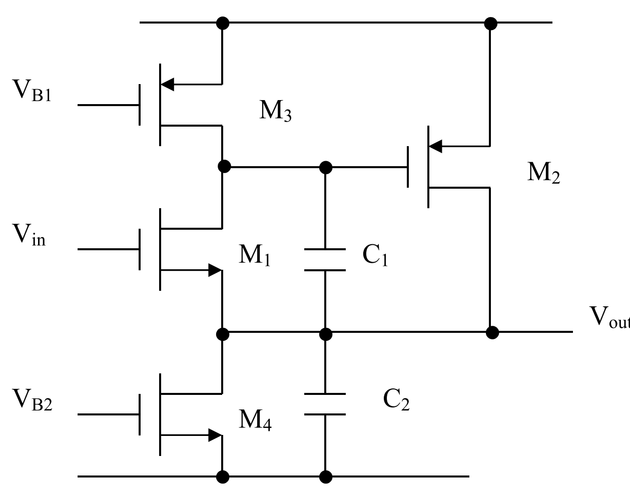
(c)



(d)



(e)



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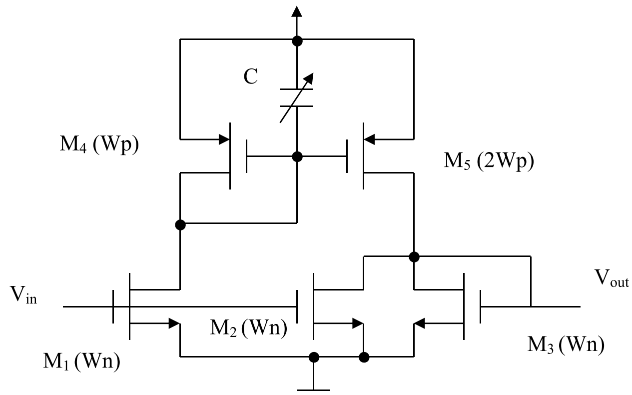

(g)

Figure 16. (a), (b) Single transistor biquads (c), , (g) first –order all-pass filters, (d) topology for Op-amp non-ideality cancellation using negative resistance, (e) A single op-amp based biquad using Twin-T network, and (f) a low-pass biquad .

VIII. CONCLUSION

In this tutorial review, the history of active filters has been surveyed. We have not covered several other design techniques such as operational simulation of RLC Ladder filters, multi-loop feedback based high-order filters. The reader may consult the various books cited in Reference section. It may be appreciated that knowledge of design techniques that have been developed over six decades is sometimes useful to employ old ideas in a modern setting to meet the current requirements of low power, low voltage, and high frequency designs. We have given only representative examples of each type of filters but there are several interesting circuits available in literature which can be found in recent books, for example [19].

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