[Manipal Journal of Science and Technology](https://impressions.manipal.edu/mjst)

Volume 2
Issue 1 *Issue 1*

Article 5

6-1-2017

Design of Reliable Controller for Twin Rotor MIMO System and Closed Loop Stability Analysis

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Research Articles

Design of Reliable Controller for Twin Rotor MIMO System and Closed Loop Stability Analysis

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Abstract

The laboratory Twin Rotor Multiple Input Multiple Output System (TRMS) serves as a model of a helicopter. Station keeping or hovering in spite of uncertainties like sensor and/or actuator failures is important in Helicopter system and in TRMS. The paper discusses the design of H infinity controller, which is made reliable for TRMS sensor and actuator failure. The desired state feedback law is constructed in terms of a positive definite solution of an Algebraic Riccati equation. Further, closed loop stability of TRMS along with the H infinity controller is tested and its robust stability bound has been obtained.

Keywords: Reliable H infinity observer controller, Twin Rotor MIMO System (TRMS), Kharitonov's polynomials, Robust stability bound

I. Introduction

TRMS is a MIMO system, which has two inputs and two outputs. Even though some simplifications are made, the important dynamic characteristics remain the same as those observed in a helicopter control system. Due to the resemblance of TRMS to the helicopter system and its simplicity, for the present research work, the TRMS is selected [1] [2]. In addition, a helicopter is a manned air vehicle. In the case of system sensor, actuator failure, human life should not be at risk due to hazardous surroundings during its operation. Hence, the system reliability is very much required. This work aims at the failure study of sensor actuator system of a helicopter using a TRMS model. The proposed control method controls the TRMS with and without the sensor, actuator fail. In this work modeling error, sensor, and actuator failure are considered the uncertainties. Since the proposed control technique takes care of these uncertainties, it would be a robust and reliable control system for TRMS. In TRMS, two feedback encoders are used to measure the two states (pitch angle and yaw angle). Reliable control means that the performance remains same, before and after the actuator or sensor failure. For this, obtaining a linear model of TRMS is very much essential. The model for TRMS is obtained using a system identification method [3].

In [4], Kalman observer is designed as an observer and further Linear Quadratic Gaussian (LQG) controller is designed for TRMS. Similarly, in [5], the Kalman observer is used for isolating the sensor failure. Chebyshev neural network (CNN) is used [6], which estimates the unknown nonlinearities, whose weights are adaptively adjusted. The methodology adopted in this paper to rectify the sensor fault occurring in TRMS is similar to this, which uses H_∞ observer using [7]. In [8], a LQG compensator is used

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How to cite this article: Vidya S. Rao, V. I. George, Surekha Kamath, Shreesha C. "Design of Reliable Controller for Twin Rotor MIMO System and closed loop Stability Analysis", *Manipal J. Sci. Tech.*, vol.2(1), 47-58, 2017.

as a state feedback inner loop controller. In [9], the Linear Quadratic Regulator (LQR) controller has been designed using the computed torque technique. In [10], Fuzzy Proportional Integral Derivative (PID) controller is designed by considering the non-linearity. In all the above papers, the controller is designed without considering the sensor failure or actuator failure. The papers [11] [12] give the H_∞ observer-controller for TRMS. The identified model is stochastic which has modeling errors, which are considered the uncertainties. Two other uncertainties on the system are the sensor failure or actuator failure or both sensor-actuator failures. Hence, the H_∞ controller technique is found suitable to control the TRMS, since it incorporates robustness in the design [13-17]. This work uses a method based on the algebraic Riccati equation (ARE) for designing robust reliable H_∞ control laws for plants with a structured uncertainty. The design method consists of incorporating information on the plant uncertainty into the ARE used for nominal H_∞ disturbance-rejection designs. In this work, the H_∞ observer-controller design and the implementation on the real TRMS is done with and without sensor, actuator failure and the results are demonstrated. Further, the results obtained using the LQG controller is compared with reliable H_∞ observer-controller results. Further, the mismatch existing between the identified model of the system and the actual plant that is going to control is termed as the uncertainties in the model. Since the experimentation is conducted with and without sensor, actuator failure, there is a possibility of large range of uncertainties. The controller should include a suitable algorithm to take care of this range of uncertainties. The motivation from this has made to find the robust stability bound for closed loop TRMS along with the reliable H_∞ controller, using the Kharitonov's stability theorem [18] [19].

II. Reliable H_∞ controller design for TRMS

The TRMS transfer functions, obtained using the system identification technique, are shown in (1) to (4). The state space representation is shown in (5).

Main pitch: (Transfer function of pitch angle to the voltage supplied to main rotor):

$$\frac{y1(s)}{u1(s)} = \frac{0.0002s^9 + 0.01569s^8 + 1.3339s^7 + 5.0689s^6 + 14.1751s^5 + 24.2433s^4 + 29.8257s^3 + 23.5613s^2 + 11.7037s + 1.8998}{s^{10} + 5.0404s^9 + 19.7749s^8 + 51.7518s^7 + 105.7788s^6 + 164.6851s^5 + 196.3385s^4 + 172.6767s^3 + 105.6574s^2 + 38.6068s + 5.2402} \quad (1)$$

Cross yaw: (Transfer function of pitch angle to the voltage supplied to tail rotor):

$$\frac{y1(s)}{u2(s)} = \frac{-0.0103s^{10} - 0.042016s^9 + 0.44703s^8 + 1.6726s^7 + 7.1798s^6 + 14.8825s^5 + 27.5294s^4 + 33.0653s^3 + 29.1328s^2 + 16.1547s + 2.7327}{s^{10} + 5.0404s^9 + 19.7749s^8 + 51.7518s^7 + 105.7788s^6 + 164.6851s^5 + 196.3385s^4 + 172.6767s^3 + 105.6574s^2 + 38.6068s + 5.2402} \quad (2)$$

Cross pitch: (Transfer function of yaw angle to the voltage supplied to main rotor):

$$\frac{y2(s)}{u1(s)} = \frac{0.048575s^9 + 0.22063s^8 + 0.70772s^7 + 1.5724s^6 + 2.5211s^5 + 2.8913s^4 + 2.3054s^3 + 1.1773s^2 + 0.3219s + 0.034081}{s^{10} + 5.0404s^9 + 19.7749s^8 + 51.7518s^7 + 105.7788s^6 + 164.6851s^5 + 196.3385s^4 + 172.6767s^3 + 105.6574s^2 + 38.6068s + 5.2402} \quad (3)$$

Main yaw: (Transfer function of yaw angle to the voltage supplied to tail rotor):

$$\frac{y2(s)}{u2(s)} = \frac{0.0016s^{10} - 0.010835s^9 + 0.08862s^8 + 0.70031s^7 + 2.784s^6 + 8.0362s^5 + 15.8111s^4 + 23.5471s^3 + 24.6452s^2 + 16.1658s + 4.8995}{s^{10} + 5.0404s^9 + 19.7749s^8 + 51.7518s^7 + 105.7788s^6 + 164.6851s^5 + 196.3385s^4 + 172.6767s^3 + 105.6574s^2 + 38.6068s + 5.2402} \quad (4)$$

$$\boldsymbol{A} = \begin{bmatrix} -1.1930 & -2.1415 & -1.7570 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.9204 & -1.5760 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.6760 & -1.1610 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.2510 & -0.7983 & -0.8150 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.25 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & -0.0103 \\ 0 & 0.0016 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0002 & 0.0075 & 0.637 & 0 & 0 & 0.0099 & 0.6174 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1943 & 0.0410 & 0 & 0 & -0.0189 & 0.1286 & 0.7607 \end{bmatrix} \tag{5}$$

First step in designing the reliable controller is to design the H_∞ observer. Using the game theory approach, dynamic real time H_∞ observer is designed with the goal of finding the correct observer gain K_0, which minimizes the difference between the predicted output and the true output. Here, by varying the observer gain the H-infinity observer decides which output to place more emphasis on. Its task is to place less emphasis on the noisy measurements and more on the actual measurements. While designing the H_∞ observer, $P(0)$and X(0)have been assumed as identity and zero, respectively [3]. The H_∞ observer design equations are given in [7] [20].

For designing the H_∞ observer based reliable controller, the design equation given in [21] is followed.

For the implecation the values for the Y= 0.01, $\bar{a}_j$=1, $\underline{a}_j$ = 0. α = 0.5 for 50 percent actuator failure.

III. Reliable H_∞ controller implementation for TRMS

In this part of the paper, the validation of H_∞ controller designed is done on TRMS setup with and without the failure of sensor and actuator.

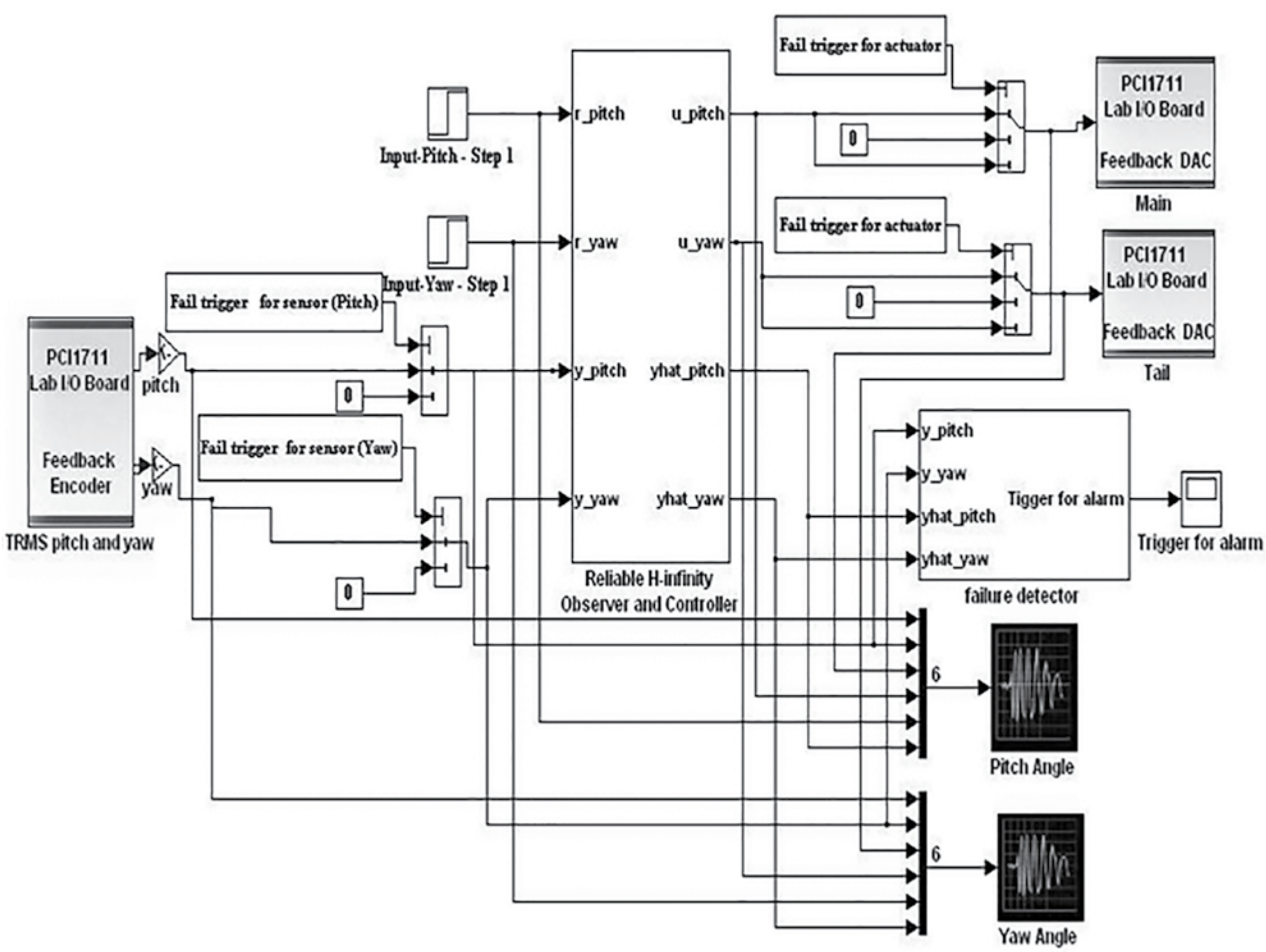

Figure 1: Reliable H_∞ controller for TRMS

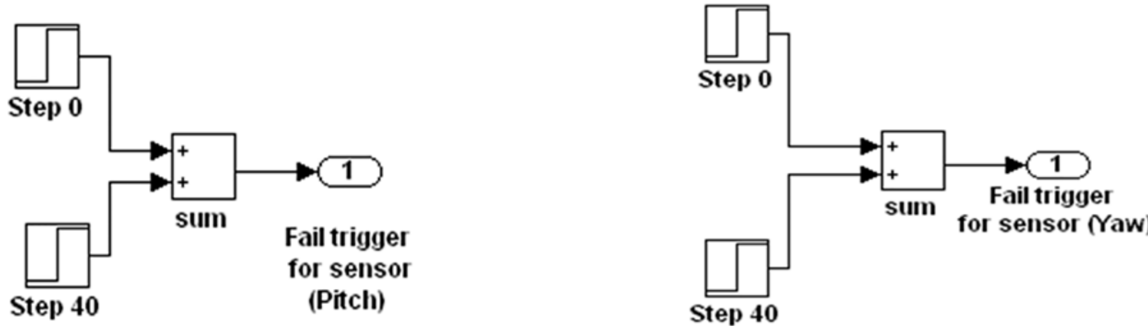

Figure 2a: Sensor Fail trigger circuit

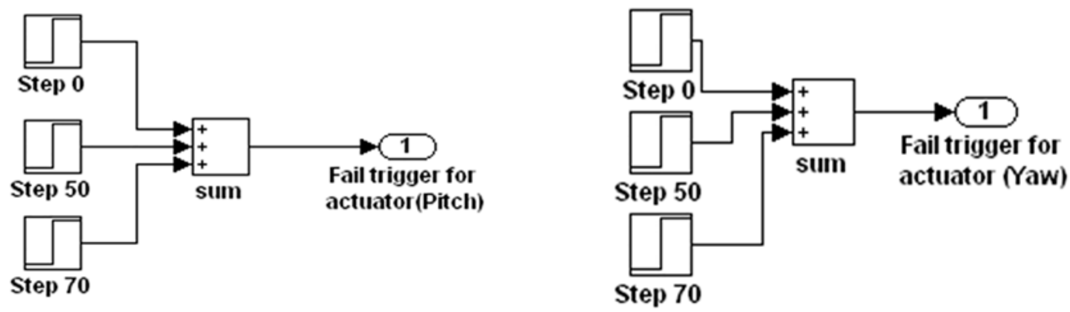

Figure 2b: Actuator Fail trigger circuit

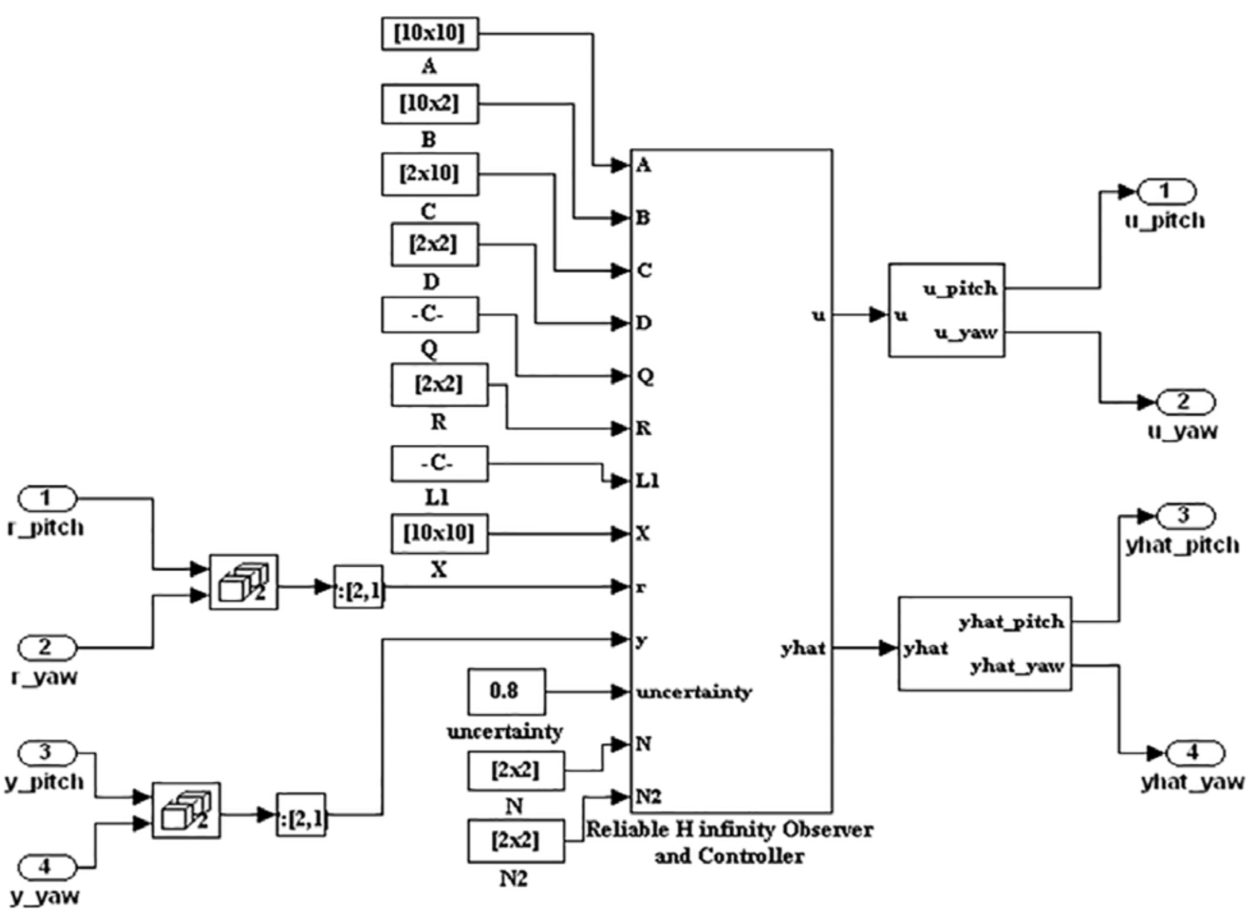

Figure 3: A Simulink diagram for H-infinity observer and controller

Simulink block diagram for the control of TRMS with sensor-actuator failure is shown in Figures. 1 to 8. The sensor has failed at t=40s and the actuator at t=50s. The simulink diagram for reliable H_∞ controller for TRMS is shown in Figure 1, the sensor fails trigger circuit, and the actuator fail trigger circuits are respectively shown in Figure 2a and Figure 2b.

The r_pitch and the r_yaw are the references for pitch input and yaw inputs respectively, given as step input with amplitude 1 radian. The feedback encoder measures y_pitch and y_yaw from the Controller, the control inputs u_pitch and u_yaw, are fed to the DAC systems as shown in Figure 1. The observer outputs from the H_∞ observer, are yhat_pitch and yhat_yaw.

The Figures. 2 to 8 show the subsystems for implementing reliable H_∞ observer and controller for TRMS. Failure detector subsystem is shown in Figure 3.8, the error of estimated pitch and yaw are minimized. Trigger for alarm will appear, if the difference between them is beyond 0.05 radian for more than 5s, which warns that there is a failure in the sensor and/or actuator.

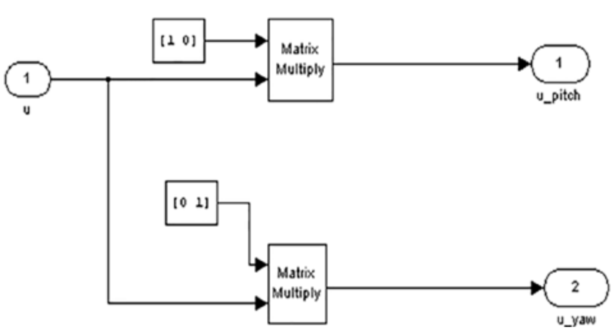

Figure 4: Pitch and yaw control input

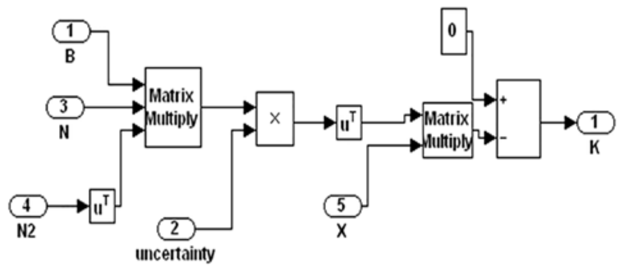

Figure 5: Circuit to calculate controller gain, K

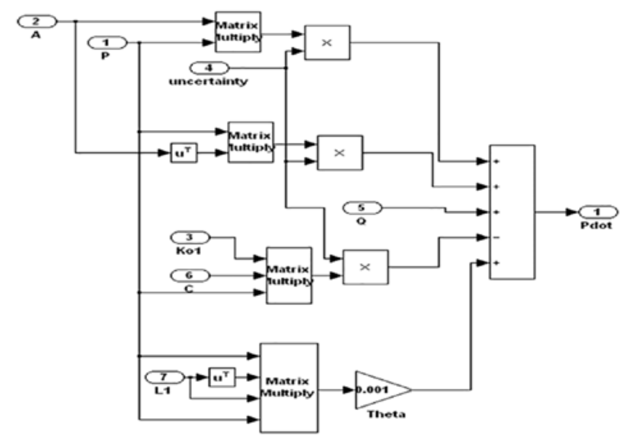

Figure 6: A simulation diagram for the solution of the Riccati equation

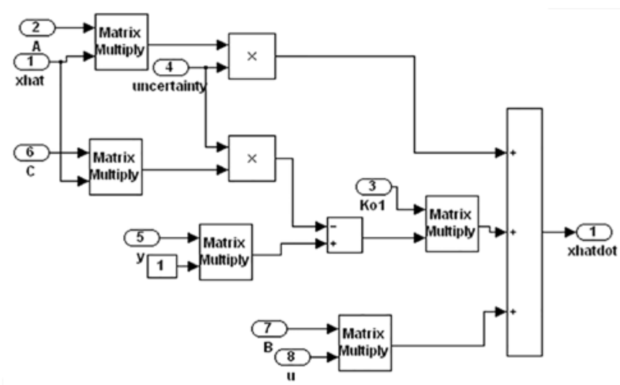

Figure 7: Block diagram representation estimated state (xhat)

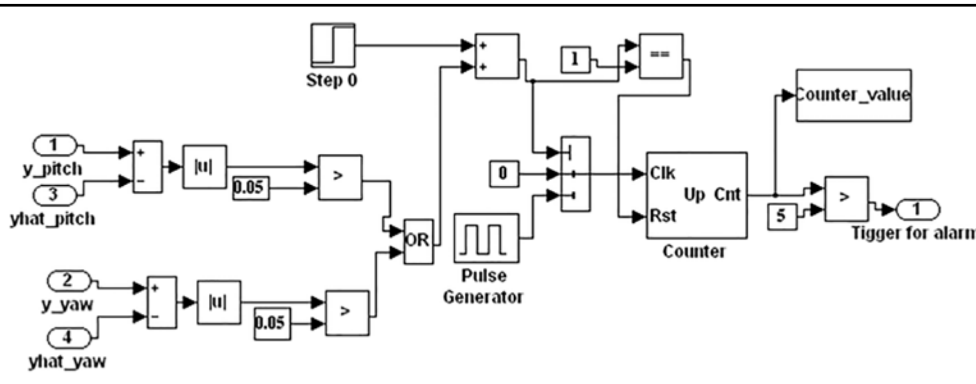

Figure 8: Failure analysis system

IV. LQG controller Design for TRMS

LQG controller is designed for TRMS [24] whose block diagram is shown in Figure 9.

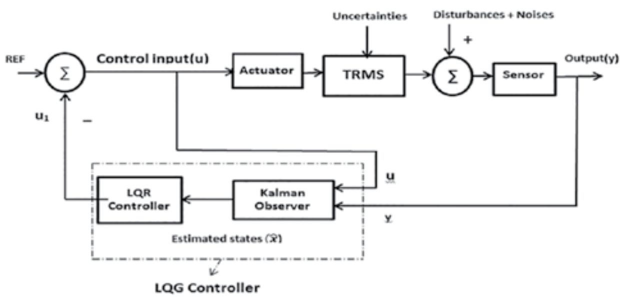

Figure 9: TRMS and LQG controller

In this paper, the control signal μ is generated. LQG controller is the combination of Kalman observer and LQR. The goal of LQR is to find the control sequence u_1, which minimizes a quadratic cost on the states and inputs as shown in (6).

$$J_{LQR} = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} x^T Q_1 x + u_1{}^T R_1 u_1 \quad \text{-(6)}$$

Where, x is the state vector, Q_1 and R_1 are the positive definite matrices, which are weighting matrix on the states and yields optimal gain K_1 for states [22] [23] [24]. The optimization of the cost function gives the optimal signal as in (7)

$$u_1 = K_1 \hat{x} \quad \text{- (7)}$$

Where $\hat{x}$ are the states of TRMS estimated by Kalman observer and

$$K_1 = R_1{}^{-1} B^T P \quad \text{- (8)}$$

P is found by solving the Riccati equation shown in (9)

$$A^T P + PA - PBR_1{}^{-1} B^T P + Q_1 = 0 \quad \text{-(9)}$$

The TRMS model has ten states with only two states being measurable, necessitating the inclusion of an observer, which is done by Kalman observer. The Kalman observer gain matrix K_0 and the controller gain is K_1 computed using (8). Combining the state feedback with the estimation problems, the LQG control signal μ is obtained as in (10).

$$u = REF - u_1 \quad (10)$$

V. Simulation Results of TRMS

A. Simulation Results

i. Under no sensor and actuator failure

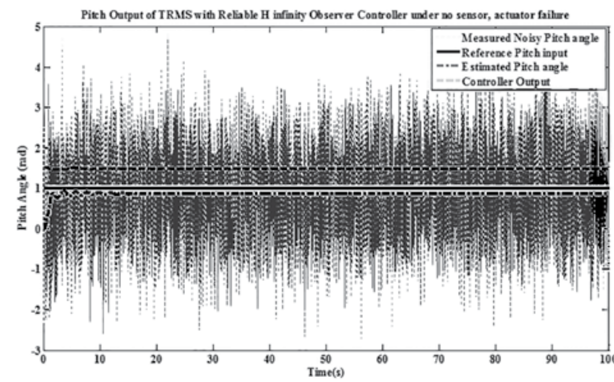

Figure 10a: TRMS pitch output with no sensor and actuator failure

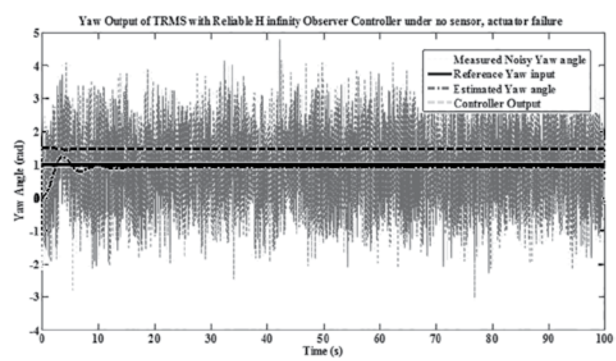

Figure 10b: TRMS yaw output with no sensor and actuator failure

As depicted in Figures. 10a and 10b, the sensors and the actuators of TRMS are not faulty. The actual pitch and yaw angles are same as that of the reference pitch and the yaw angles.

ii. Under sensor and actuator failure

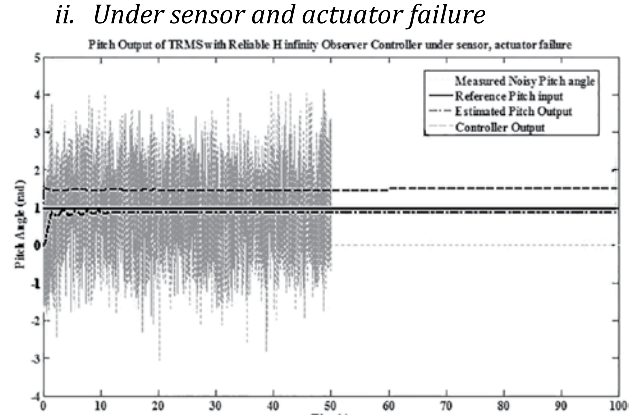

Figure 11a: TRMS pitch output with failure of both sensor and actuator

In this case, the estimated state, $\hat{x}$ (estimated using H_∞ observer) is fed to the H_∞ controller. This results in an observer based reliable control system, which can tolerate sensor and actuator failure, as shown in Figures. 11a and 11b. At t=50s the TRMS sensor and at t=60s the TRMS actuator are failed. Due to reliable H_∞ observer controller, the controller output is unaffected even after the failure of sensor and actuator.

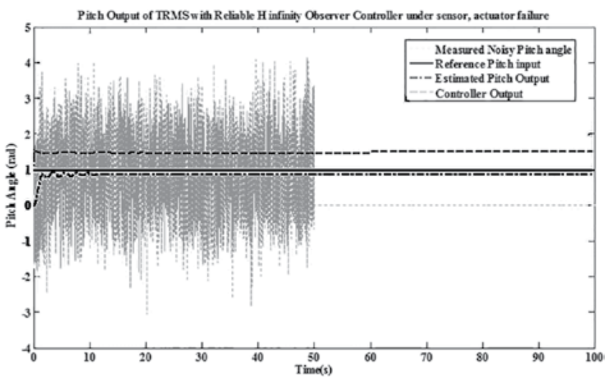

Figure 11b: TRMS yaw output with failure of both sensor and actuator

B. Implementation results

i. Under no sensor and actuator failure

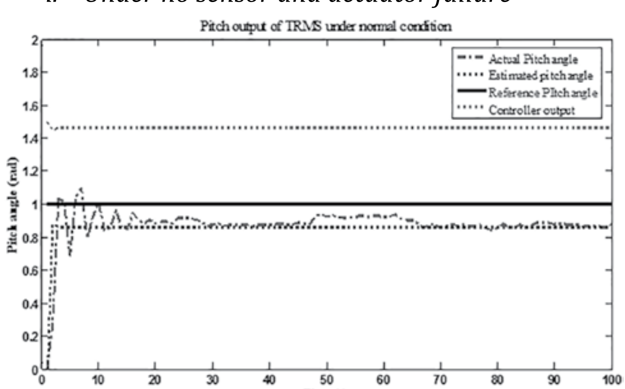

Figure 12a: TRMS pitch output under no sensor actuator failure

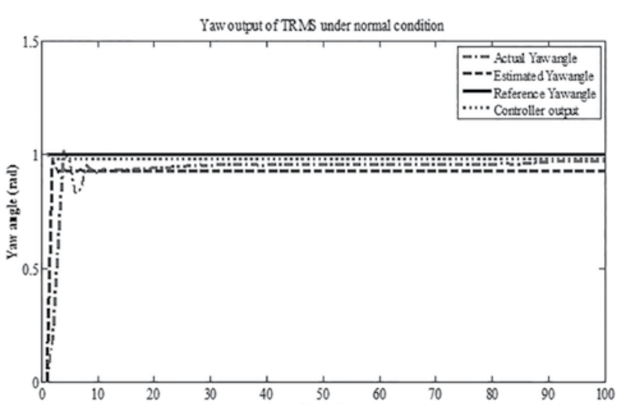

Figure 12b: TRMS yaw output under no sensor actuator failure

Without any failure of the sensor or actuator of TRMS, the pitch angle and yaw angle of TRMS are controlled and are shown in Figures. 12a and 12b. The failure detector output will be zero, since the sensor or actuator is in normal operational conditions.

ii. Sensor/actuator both failure

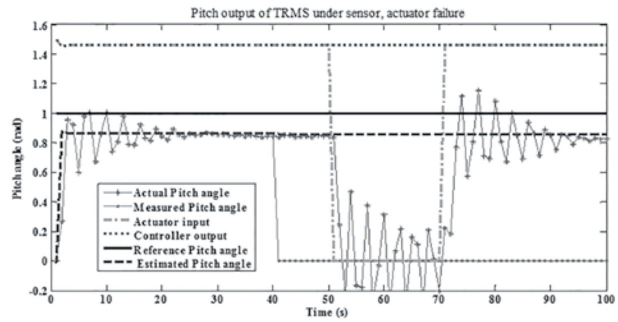

Figure 13a: TRMS pitch angle under both sensor and actuator failure

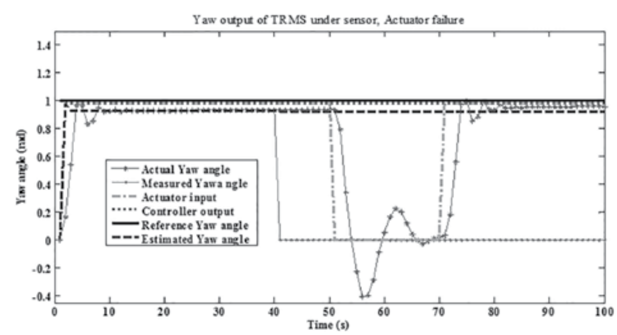

Figure 13b: TRMS yaw angle under both sensor and actuator failure

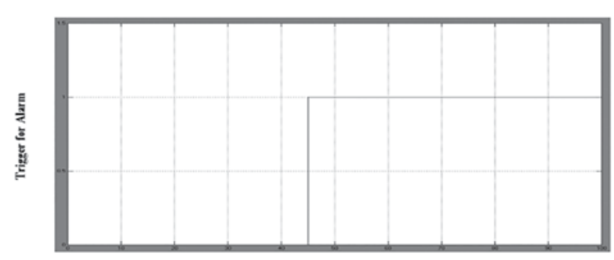

Figure 13c: Failure Detector output/sensor and actuator failure

The sensor of TRMS is failed at t=40s shown in Figures. 13a and 13b. Since the measurement system fails, the measured pitch and the yaw angles become zero. However, there is a control signal present, which proves the efficiency of H_∞ observer controller. The actuator failure occurs at t=50s. The output reduces to zero since there is no driving source. However, even in this case the control signal is present proving that the designed H_∞ observer controller is reliable.

The Figure 13c shows the failure detector output. The TRMS output is brought to the reference level, when the actuator is replaced back (at t=70s).

VI. Results of LQG Controller for TRMS

A. LQG control under no sensor, actuator failure

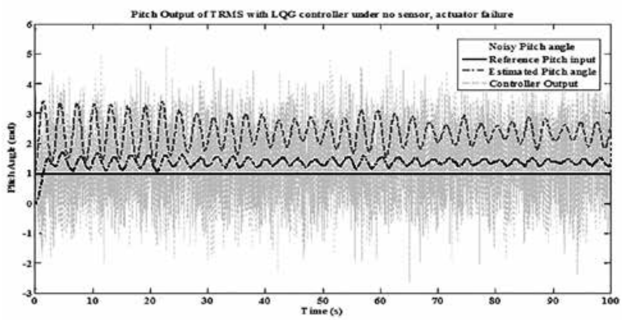

Figure 14a: TRMS pitch output under no sensor, actuator failure

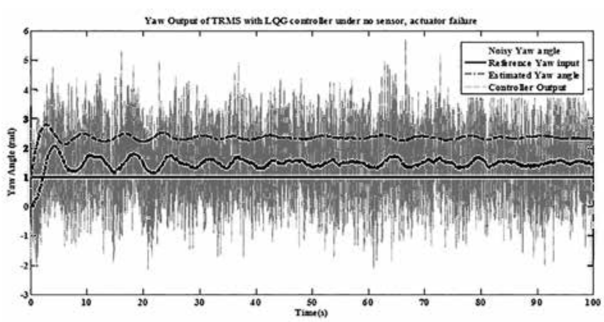

Figure 14b: TRMS yaw output under no sensor, actuator failure

B. LQG control under sensor, actuator failure

i. Simulation Results

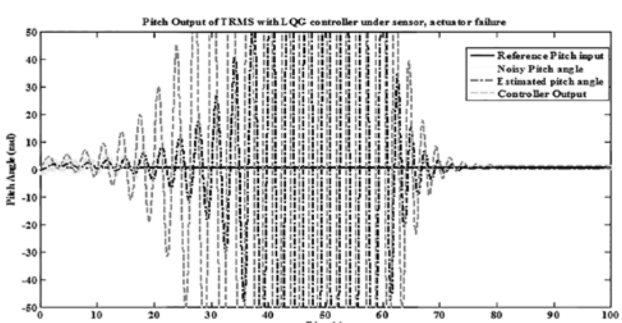

Figure 15a: TRMS pitch output under both sensor, actuator failure

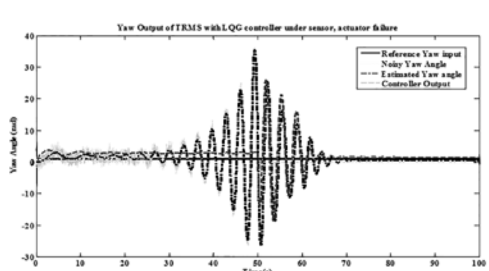

Figure 15b: TRMS yaw output under both the sensor and the actuator failure

It is seen from Figures. 14a and 14.b, when the sensor and the actuator are working fine, the LQG controller gives the stable pitch and yaw output as in the case of H_∞ observer controller (Figures.10a and10b). However, the LQG controller gives the oscillating output as depicted from Figures. 14a and 14b. This proves that its performance is lower than the performance of H_∞ observer controller for TRMS. Figures. 15a and 15b demonstrate that the pitch angle and the yaw angle of TRMS go out of control after sensor actuators of TRMS have failed. The controller output after 50s is erratic.

ii. Implementation Results

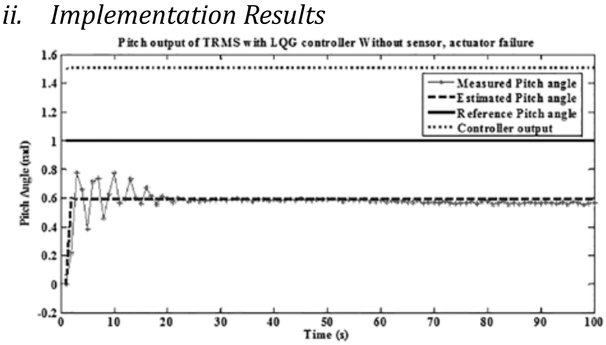

Figure 16a: TRMS pitch output under no sensor, actuator failure

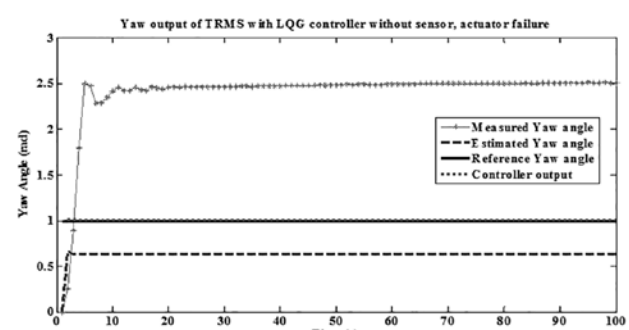

Figure 16b: TRMS yaw output under no sensor, actuator failure

C. A comparison of LQG controller and reliable H_∞ observer controller designed for TRMS

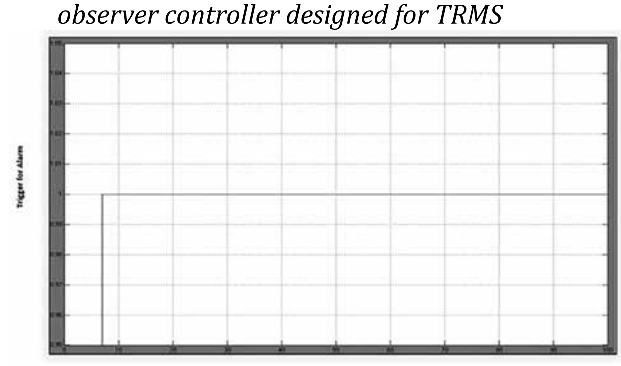

Figure 16c: Output of failure detector under no sensor, actuator failure

The performance of LQG controller from Section 6 is compared with the performance of H_∞ controller from Section 5 which is designed for the TRMS. The inferences drawn are shown in Table 6.2.

VII. Stability Analysis of TRMS with reliable H_∞ controller using Kharitonov's stability theorem

The robust stability bound is the stability margin within which the closed loop TRMS along with H_∞ controller is stable. Since the TRMS is a MIMO system, the system matrix A varies with the uncertainties for which the H_∞ controller has to be made robust by finding a suitable controller gain K, which also varies with the uncertainties. To make the TRMS robustly stable after designing the controller for TRMS, the percentage of sensor or actuator failure is varied within a certain interval. That is the variation in TRMS state space parameters *A,B,C,D* is made and an allowable range before it loses its stability is observed. By trial and error approach, the actual TRMS state space parameters are varied from 0.25 to 1.75 taken from (1) to (5). This means the uncertainty is varied between 25% of actual parameters on lower side to 75% more than the actual parameters on the upper side. During experimentation on TRMS, it is found that uncertainty is 0.5 to 1.27 of actual TRMS parameters for which a closed loop TRMS along with H_∞ observer controller is stable. These are a range of model parameters and beyond which the TRMS model is not valid. Using these uncertainty limits, the characteristic equation of the closed loop TRMS is formed with H_∞ controller, which gives the interval polynomial for TRMS as shown in (11) and (12). Using this interval polynomial (along with the uncertainties) four Kharitonov's polynomials are formed [25-30] as shown in (13) to (16).

Table 1: A comparison between the performances of LQG controller and the reliable H_∞ controller

Type of analysis	LQG controller	Reliable H_∞ controller
Simulation	i . Pitch and yaw output are oscillatory. (Figs. 14a, 14b)	i. Pitch and yaw output are constant. (Figs. 10a, 10b)
	ii. Controller output is oscillatory (without sensor, actuator failure) and erratic (with sensor actuator failure). (Figs. 14a, 14b, 15a, and 15b)	ii. Controller output is same and constant (with and without sensor, actuator failure). (Figs. 10a, 10b, 11a, and 11 b)
Implementation	i. Very large steady state error exists in pitch as well as yaw output (without sensor, actuator failure) (Figs. 16a, 16b).	i . Very small steady state error exists in pitch as well as yaw output (without sensor, actuator failure). (Figs. 12a, 12b)
	ii. With sensor, actuator failure the algorithm completely fails.	ii. With sensor, actuator failure the algorithm works same as that of without failure. (Figs. 13a, 13 b)

$$0.1s^{10} + 1.45s^9 + 10.65s^8 + 47.2s^7 + 169.1s^6 + 246.6s^5 + 270.7s^4 + 341.9s^3 + 173s^2 + 0.005s + 0.0005 = 0 \quad (11)$$

$$0.1s^{10} + 1.48s^9 + 11.84s^8 + 48.7s^7 + 138.9s^6 + 332.2s^5 + 422.1s^4 + 169.98s^3 + 45.77s^2 + 49.8s + 5.7 = 0 \quad (12)$$

$$K_{h1}(s) = 0.1s^{10} + 1.45s^9 + 10.65s^8 + 48.7s^7 + 138.8s^6 + 246.6s^5 + 270.7s^4 + 169.9s^3 + 45.8s^2 + 0.005s + 0.0005 = 0 \quad (13)$$

$$K_{h2}(s) = 0.1s^{10} + 1.47s^9 + 11.8s^8 + 47.2s^7 + 169.1s^6 + 332.2s^5 + 422.1s^4 + 341.9s^3 + 173s^2 + 49.8s + 5.74 = 0 \quad (14)$$

$$K_{h3}(s) = 0.1s^{10} + 1.45s^9 + 11.84s^8 + 48.8s^7 + 169.1s^6 + 246.6s^5 + 422.1s^4 + 169.9s^3 + 173s^2 + 0.005s + 5.74 = 0 \quad (15)$$

$$K_{h4}(s) = 0.1s^{10} + 1.47s^9 + 10.65s^8 + 47.2s^7 + 138.9s^6 + 332.2s^5 + 270.7s^4 + 341.9s^3 + 45.8s^2 + 49.8s + 0.0005 = 0 \quad (16)$$

The roots of four Kharitonov's polynomials (13) to (16) are found and are tabulated in Table 7.1. It is found that any of the polynomials do not have Right Hand Side pole. That is, all the Kharitonov's polynomials have roots, which are negative and hence closed loop TRMS along with H_∞ observer controller for the variation in the coefficients of the characteristic polynomial as in (11) and (12) with the uncertainty bound 0.5 to 1.27 is stable. Thus, the Kharitonov's Stability Theorem is successfully applied and is verified for closed loop TRMS with H_∞ observer controller. The robust stability bound for TRMS is obtained and is shown in Table 7.2.

Table 2: Roots of Kharitonov's polynomials of TRMS

No	Kharitonov's polynomials formed for TRMS	Roots of Kharitonov's polynomials formed
K_{h1}	$0.1s^{10} + 1.45s^9 + 10.65s^8 + 48.7s^7 + 138.8s^6 + 246.6s^5 + 270.7s^4 + 169.9s^3 + 45.8s^2 + 0.005s + 0.0005 = 0$	-2.0716 ± 4.1491i -4.0056 -0.9310± 1.0266i -1.9891 -1.6652 -0.8349 -0.0001± 0.0033i
K_{h2}	$0.1s^{10} + 1.47s^9 + 11.8s^8 + 47.2s^7 + 169.1s^6 + 332.2s^5 + 422.1s^4 + 341.9s^3 + 173s^2 + 49.8s + 5.74 = 0$	-5.5413 ± 5.3116i -0.3798± 3.5644i -0.5357± 0.8840i -0.4479± 0.4715i -0.6200 -0.2707
K_{h3}	$0.1s^{10} + 1.45s^9 + 11.84s^8 + 48.8s^7 + 169.1s^6 + 246.6s^5 + 422.1s^4 + 169.9s^3 + 173s^2 + 0.005s + 5.74 = 0$	-0.2638± 5.6789i -1.2575± 1.3719i -2.1484± 0.1672i -1.0976 -0.5901 -0.2364 ±0.3377i
K_{h4}	$0.1s^{10} + 1.47s^9 + 10.65s^8 + 47.2s^7 + 138.9s^6 + 332.2s^5 + 270.7s^4 + 341.9s^3 + 45.8s^2 + 49.8s + 0.0005 = 0$	-6.0258 -3.7645± 4.0395i -0.3681± 3.4325i -0.2494± 1.1083i -0.0051±0.4198i -0.0001

Table 3: Range of TRMS model parameter variation (Robust stability bound of TRMS)

Parameter		Minimum	Nominal	Maximum
TRMS characteristic polynomial	(coefficients of s^{10})	0.1	0.1	0.1
	(coefficients of s^9)	1.45	1.47	1.48
	(coefficients of s^8)	10.45	10.99	11.84
	(coefficients of s^7)	47.2	47.6	48.7
	(coefficients of s^6)	138.9	142.8	169.1
	(coefficients of s^5)	246.6	272.1	332.2
	(coefficients of s^4)	270.7	302.1	422.1
	(coefficients of $s^{3)}$	169.98	198.2	341.9
	(coefficients of s^2)	45.77	105.8	173
	(coefficients of s^1)	0.005	38.4	49.8
	(coefficients of s^0)	0.0005	3.9	5.7

VII. Conclusion

The reliable H_∞ controller is designed for the TRMS without and with sensor, actuator failure. The simulation and the implementation results show that even if the sensor, actuator of the TRMS fails the TRMS remains stable with the H_∞ controller. The reliable H_∞ controller performance is compared with the LQG control technique and proved that the reliable H_∞ controller performs superior to the LQG controller for the TRMS. Further, the range of robust stability bound for the closed loop TRMS along with the reliable H_∞ controller using Kharitonov's Stability Theorem is found. The variation in parameters of TRMS from its nominal values is shown. The stability analysis proves that within the mentioned uncertainty limit the TRMS along with the reliable H_∞ controller gives the closed loop stable response.

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