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A Note on Algebraic Structures

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Research Articles

A Note on Algebraic Structures

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Abstract

In this paper, we introduce the idea of 2-Banach algebra and 2-C*-algebra with suitable examples and some related results are established. We provide the recent developments and related results in near-algebra. We also give a generalization of gamma nearring and a module over a gamma nearring with suitable illustration.

Keywords: Linear 2-normed spaces, Banach algebra, C*-algebra, Nearring, gamma nearring, AMS Classifications: 41A65, 41A15, 16Y30

Banach algebra

Definition 1 [8]: Let X be a vector space over $R$with $dim(x) \geq 2$and ||.,.|| be a mapping from X × X to R, which holds the following conditions: for all u, v, w∈X and β∈R

(N1) ||u,v||=0if x and y are linearly dependent
(N2) ||u,v||=||v,u||
(N3) ||βu,v||=|β|||u,v||
(N4) ||u+v,w||≤||u,w||+||v,w||

then the map ||.,.|| is called a 2-norm on X. The pair (X,||.,.||)) is called a 2- normed space.
Some of the basic properties of 2-norms are,

1. ||u,v||≥0 for every u,v∈X.
2. ||u,v+βu||=||u,v|| for all u and v in X and for every β∈R
3. Let X be a 2-normed space then X is a locally convex topological vector space.
4. For a fixed c∈X,P_c (u)=||u,c||, for u ∈ X is a semi norm and the family {P_c;c∈X} of semi norms form a locally convex topology on X.

2000 *Mathematics Subject Classification.* 41A65, 41A15.

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Key words: Linear 2-normed spaces, Banach algebra, accretive operators, nearring.

Example 2: Let $X=R^2$ define ||u,v|| area of the parallelogram determined by the vector u and v as the adjacent sides.
R^3 is a 2-normed space equipped with 2-norm

$$\|u, v\| = \begin{vmatrix} i & j & k \\ x & y & z \\ s & t & w \end{vmatrix}$$

where $u= (x,y,z)$ and $v = (s,t,w)$.

Example 3: Let X be a normed linear space then

$$\|T, S\| = \frac{1}{2}\left\{\begin{vmatrix} T(x)S(x) \\ T(y)S(y) \end{vmatrix}; T, S \in X \right\}$$

defines a 2-norm on X, where X^* is the set of all bounded linear functionals on X with norm ≤ 1.

Definition 4 [8]: Let (X, ‖., .‖) be a 2-normed space, and $\{x_n\}$ be a sequence of X. Then the sequence $\{u_n\}$ is said to be convergent to x if $\|u_n - u, z\| \to 0$ as $n \to \infty$ for all z ∈ X.

Definition 5 [8]: Let $(X, \|., .\|)$ be a 2-normed space then a sequence $\{u_n\} \in X$ is said to be a *Cauchy sequence* if $\|u_m - u_n, z\| \to 0$ as $m, n \to \infty$for all $z \in X$.

Definition 6: [9] An operator T: $D(T) \subset X \to X$ is said to be accretive if for every $z \in D(T)$ $\|u - v, z\| \leq \|(u - v) + \lambda(Tu - Tv), z\|$ for all $u,v \in D(T)$ and $\lambda > 0$.

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Definition 7: [9] An operator T: $D(T)\subset X\rightarrow X$ is said to be m-accretive if $Range$ $(I+\lambda T) = X$ for $\lambda > 0$.

Definition 8 [8]: A2-normed space is said to be *complete* if every Cauchy sequence in X is convergent to a point of X.

A Complete 2-normed space is called *2- Banach space*.

Example 9 [8]: Let B_n denote the set of all real polynomials of degree $\leq n$ in the interval [0, 1]. By considering the usual addition and scalar multiplication, B_n is a linear space over R. Let $\{x_0, x_1, x_2,...x_{2n}\}$ be distinct fixed points in [0, 1] and define 2-norm on B_n by

$$\|u, v\| = \sum_{k=0}^{2n} u(x_k)v(x_k) \vee$$

where u and v are linearly independent and equal to zero otherwise. Then $(B_n, ||, ||)$ is a 2-normed space.

Then B_n is a 2-Banach space.

Consider an accretive operator T: $D(T) \subset X\rightarrow X$ then for $\lambda>0$, $(I+\lambda T)^{-1}$ exists. We have, T: $D(T) \rightarrow Range(T)$ then $(I + \lambda T)$ is onto. Let $u,v \in D(T)$ with $(I + \lambda T)u = (I + \lambda T)\, v$ implies $(u - v) + \lambda(Tu - Tv) = 0$ implies $\|(u\text{-}v) + \lambda(Tu - Tv), z\| = 0$ for every $z \in X$.Since T is accretive, we have $\|u\text{-}v,z\| \leq 0$for every $z \in X$ implies $\|u\text{-}v,z\| = 0$ for every $z \in X$ implies $u\text{-}v = 0$ implies $u=v$. So $(I + \lambda T)$ is one-one. Hence for $\lambda > 0$, $(I + \lambda T)^{-1}$ exists.

Definition 10: Let $(X, \|.,.\|)$ be a linear 2- normed space then an operator T on X is said to be non-expansive if, and only if, or each u, v $\in$ D(T), $\|Tu - Tv,\ z\| \leq \|u\text{-}v,\ z\|$ for every z $\in$ X. T is said to be expansive if for each u, v $\in$D(T), $\|Tu - Tv,\ z\| > \|u\text{-}v,z\|$ for every z $\in$ X.

Definition 11: Let $(X, \|.,.\|)$ be a linear 2- normed space, a subset L of X of the form $\{u+\ tv;\ t \in R^{+}\}$ where $u, v \in X$ and $v\neq 0$, is called a line.

Example 12: Let$(X,\|.,.\|)$ be a linear 2-normed space and S be a subset of $L=\{x+\alpha y:\ \alpha \in R^{+}\}$. Define F: $S\rightarrow X$ by $F(x+\alpha y)=(\alpha/(\alpha+1))y$ for x, y $\in$ X, Z $\in$ X is a non-expansive mapping on X.

Example 13: Let $(X,\|.,.\|)$ be a linear 2-normed space and S be a subset of $L=\{x+\alpha y:\ \alpha \in R^{+}\}$. Define F: $S\rightarrow X$ by $F(x+\alpha y)=(\alpha/(1+\|y,z\|))y$ for x, y$\in$ X, z $\in$ X, is a non- expansive mapping on X.

Proposition 14: *Let* $(X, \|.,.\|)$ *be a linear 2- normed space and* $T:D(T) \subset X\rightarrow X$ *be an operator. If* $(I+\lambda T)$ *is expansive for all* $\lambda>0$ *then T is accretive.*

Remark 15: If T is an accretive operator on a2-normed space X then $(I+\lambda T)^{-1}$ is non-expansive.

Definition 16: Let $(X, \|.,.\|)$ be a linear 2- normed space F be a non-empty subset of X and $t \in F$ then F is said to be e-bounded if there exists some $K > 0$ such that $\|u, t\| \leq K$ for all $u \in F$. If for all $t \in F$, F is t-bounded then F is called a bounded set.

Let $(X, \|.,.\|)$ be a linear 2- normed space and T be an m-accretive operator on X. For n=1,2,3, ... Define the resolvent of T as, $J_n(u) = (I + n^{-1}T)^{-1}(u)$ and the Yosida approximation $T_n(u) = n(I - J_n)(u)$ for all $u \in X$ and $\lambda > 0$.

Proposition 17:*Let* $(X, \|.,.\|)$ *be a linear 2- normed space and T be a m-accretive operator then*

(i) $\|J_n u - J_n v, z\| \leq \|(u\text{-}v), z\|$

(ii) $\|T_n u - T_n v, z\| \leq 2n\|(u\text{-}v), z\|$ *for all u,v∈X*

Proposition 18: *Let* $(X, \|.,.\|)$ *be a linear 2- normed space and F be a non empty bounded subset of X and* T: $F\rightarrow F$ *be an accretive operator then there exists some* $M > 0$ *and for* $x \in F$, $\|T_n u, z\| \leq M$ *for all* $z \in E$.

Proposition 19: *Let* $(X, \|.,.\|)$ *be a linear 2- normed space and F be a non empty bounded subset of X and* T: $F\rightarrow F$ *be an accretive operator. If* $u \in F'$ *then* $J_n u \rightarrow u$.

Definition 20: Let $(X, \|.,.\|)$ be a linear 2- normed space then the mapping T: $X \rightarrow X$ is said to be a contraction if there exists some $k \in (0, 1)$ such that $\|Tu - Tv, z\| \leq k\|u\text{-}v, z\|$ for all u,v,z $\in$ X.

Lemma 21: *Let* $(X, \|.,.\|)$ *be a linear 2- normed space then every contraction* T: $X \rightarrow X$ *is sequentially continuous.*

Definition 22: Let $(X, \|.,.\|)$ be a 2-Banach space of dimension greater than 2 then X is said to be a 2-Banach algebra if X is a real algebra and for all mutually independent elements $x, y, z \in X$, it must satisfy the condition

$$\|xy,z\| \leq k\|x,z\|\|y,z\|$$

for some positive number k.

Definition 23: A 2-Banach algebra $(X, \|.,.\|)$ is said to have an identity element e if $e.x = x.e = x$ for all $x \in X$ and $\|x, e\| \neq 0$.

Example 24: Let R^3 be a vector space of dimension 3. Let $a = a_1e_1 + a_2e_2 + a_3e_3$ and $b = b_1e_1 + b_2e_2 + b_3e_3$ where (e_1, e_2, e_3) is the standard basis in R^3. Define

$$\|a, b\| = [(a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2]^{\frac{1}{2}}$$

then $(R^3, \|.,.\|)$ is a 2-Banach algebra.

Example 25: Let X be a commutative Banach algebra and S be the set of all positive multiplicative functionals f on X with $\|f\| \leq 1$. For each $x, y \in X$. Define

$$f(x)g(y) - g(x)f(y) \vee_{f,g \in S}$$
$$\|a, b\| =$$

then $(X, \|.,.\|)$ is a 2-Banach algebra.

In a 2-Banach algebra E, for all $x, x', y, y', z \in E$

$$\|(xy - x^{\wedge\prime} y^{\wedge\prime}), z\| = \|x(y - y^{\wedge\prime}), z\| + \|(x - x^{\wedge\prime})y, z\|$$
$$\leq \|x(y - y^{\wedge\prime}), z\| + \|(x - x^{\wedge\prime})y, z\|$$
$$\leq k(\|x,z\|\|y - y^{\wedge\prime}, z\| + \|x - x^{\wedge\prime}, z\|\|y,z\|)$$

Definition 26: Let X be a 2-Banach algebra with identity element e. An element $r \in X$ is said to be a regular element if there exists an element $s \in X$ such that $rs = e$. An element in X which is not regular is called a singular element. In X, the set of all regular elements by R and singular elements by S.

Lemma 27: *Let X be a 2-Banach algebra with identity element e, then every element $r \in E$ such that $\|e - r, z\| < 1$ for all $z \in X$ is regular.*

Proof. Let $x = e - r$ and $y_0 = e$. Define the sequence $\{y_n\}$ by $y_{n+1} = y_0 + xy_n$ for $n = 1, 2, 3, ...$

Then, for all $z \in X$,

$$\|y_{(n+1)} - y_n, z\| = \|xy_n - xy_{n-1}, z\|$$
$$\leq k\|x, z\|\|y_n - y_{n-1}, z\|$$
$$\leq k^2 \|x,z\|^2 \|y_{n-1} - y_{n-2}, z\|$$
$$\leq \|x,z\|^n \|y_1 - y_0, z\|$$

hence, $\|y_{n+p} - y_n, z\| \leq k^n(\|x,z\|^{n+p} + ... + \|x,z\|^{n+1})$ for any integer p.

Therefore, $\{y_n\}$ is a Cauchy sequence in E. As E is complete, there exits an element $y \in X$ such that

$$\lim_{n\to\infty} y_n = y$$

We have,

$$\lim_{n\to\infty} y_{n+1} = y_0 + x \lim_{n\to\infty} y_n \Rightarrow y = e + (e - r)y \Rightarrow ry = e.$$

Definition 28: Let $(X, \|.,.\|)$ be a 2-Banach algebra. An involution on X is a mapping $*: X \to X$ satisfying the following conditions:

1. $(\alpha x + \beta y)^* = \bar{\alpha} x^* + \bar{\beta} y^*$ forall $x, y \in X$ and $\alpha, \beta \in K$
2. $(xy)^* = y^* x^*$ for all $x, y \in X$
3. $x^{**} = x$ for all $x \in X$

A real 2-Banach* algebra is a real 2-Banach algebra $(X, \|.,.\|)$ equipped with an involution * defined on it.

Example 29: Consider the real 2- Banach algebra R^3 together with the 2-norm defined in the example (1.4). Define a map $*: R^3 \to R^3$ by $(a_1, a_2, a_3)^* = (a_2, a_1, a_3)$ for all $(a_1, a_2, a_3) \in R^3$ then * is an involution on R^3. Hence R^3 is a real 2-Banach* algebra.

Definition 30: A real 2-C* algebra is a real 2-Banach* algebra X such that $\|x^*x, z\| = \|x, z\|^2$ for all $x, z \in X$.

Definition 31: Let X and Y be 2-Banach* algebra then a linear map $H: X \to Y$ is said to be a 2-Banach* algebra homomorphism if

1. $H(xy) = H(x)H(y)$ for all $x, y \in X$
2. $H(x^*) = H(x)^*$ for all $x \in X$

Definition 32: Let X and Y be 2-Banach* algebra then a linear map $D: X \to Y$ is said to be a 2-Banach* derivation if

1. $D(xy) = D(x)y + xD(y)$ forall $x, y \in X$
2. $D(x^*) = D(x)^*$ for all $x \in X$

Review of Nearrings:

Algebraic structures like semigroups, groups, and groupoids are equipped with one binary operation. However, algebraic systems with two binary operations play a vigorous role in the several areas of research digital phenomena, algebraic codes etc. One such significant algebraic structure is "nearring", which is armed with two binary operations: addition and multiplication, satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. The theory of nearrings is a sophisticated theory, which has found numerous applications in various areas. Dickson [5] showed that there exist "fields with only one distributive law" (named nearfields). These have showed up to be useful in coordinating certain important classes of geometric planes. There were links between other parts of nearrings (especially nearfields), and geometry come up at several places. It is well known that efficient block designs and codes can be constructed from finite nearrings. Many structural aspects of the theory of rings were transferred to nearrings and the new nearring precise features were discovered, constructing a theory of nearrings step by step (for example, Wedderburn – Artin Theorem for simple algebras). It is evident that every ring is a nearring. A natural example of a nearring (but not a ring) is assumed by the set M(G) of all mappings of an additively written group G (which is not essentially abelian) into G itself with usual addition of mappings: (f + g)(x) = f(x) + g(x) and composition as multiplication operation: (fg)(x) = f(g(x)) for all x∈ G, f, g ∈M(G).

Definition 33: An algebraic structure $(N, +, \times)$ is said to be a (right) nearring if it satisfies the conditions

(NR1) $(N, +)$ is a group (not necessarily abelian)

(NR2) $(N, .)$ is a semigroup

(NR3) $(a + b).c = a.c + b.c$ for all $a, b, c \in N$

In a similar way, one can define a left nearring.

Example 34:

1. Let $N=\{0,a,b,c\}$ with the binary operations defined explicitly as follows.
2. Let $(G, +)$ be an abelian group with additive identity 0. $M(G) = \{f:G\to G\}$ with $(f + g)(x) = f(x) + g(x)$ and $(f \circ g)(x) = f(g(x))$ for $f, g \in M(G)$ forms a right nearing whereas$((M(G)),+,\circ)$ is not a ring.

Addition and multiplication table

+	0	a	b	c	-	0	a	b	c
0	0	a	b	c	0	0	0	0	0
a	a	0	c	b	a	0	0	a	a
b	b	b	0	a	b	0	a	b	b
c	c	c	a	0	c	0	a	c	c

Here $(N,+)$ is a Klein's four group and $(N,+,.)$ is a nearing but not a ring since $b.a + b.c = a + b = c$ whereas $b.(a + b) = b.c = b$

3. Let $(G, +)$ be a group. Define multiplication (as trivial multiplication) on Gas $a.b = 0$ for all $a, b \in$ G. Then $(G, +, .)$ is a nearring. If tt is a non abelian group then $(G, +, .)$ is a nearrig that is not a ring.

(a) $M_0(G)=\{f\in M(G)|f(0)=0\}$

4. Let $(G, +)$ be a group (not necessarily abelian) V be a Vector space over a field F and R be a commutative ring with identity. With respect to the point wise addition + and composition ○ the following sets are nearrings.

(b) $M_c(g)=\{f\in M(G)|$ f is constant$\}$

(c) $M_{aff}(V)=\{f\in M(V)|f=g+h$ where his linear map and g is constant map$\}$

(d) $P(R)=\{f\in M(R)|$f is a polynomial function$\}$

Theorem 35: *Let* $(N, +, .)$ *be a nearring then for all* $n, n' \in N$

(i) $0n = 0$ *and* $(-n)n' = -nn'$

(ii) $-(a+b)=-b-a$ *for all* $a,b\in N$

Definition 36: Let (X,+,.) be an algebra. A mapping $||.||:X\times X\to X$ is said to be a triangular norm on X if it satisfies the following conditions; for all x,y∈X

$$||x+y||\leq||x||+||y||$$

$$||x.y||\leq\max\{||x||,||y||\}$$

$$||x||\geq 0 \text{ and } ||x||=0 \text{iff } x=0$$

Example 37: Consider the algebra G={0′, 1′, 2′, 3′} with usual addition and multiplication. Define ||a||=o(a) for every a∈G then ||.|| is a triangular norm on G

Verification:

a	b	a+b	a.b	o(a)	o(b)	o(a) o(b)	o(ab)	o(a+b)	o(a) +o(b)
0′	0′	0′	0′	0	0	0	0	0	0
0′	1′	1′	1′	0	4	0	4	4	4
0′	2′	2′	1′	0	2	0	4	2	2
0′	3′	3′	0′	0	4	0	0	4	4
1′	0′	1′	0′	4	0	0	0	4	4
1′	1′	2′	1′	4	4	16	4	2	8
1′	2′	3′	2′	4	2	8	2	4	6
1′	3′	0′	3′	4	4	16	4	0	8
2′	0′	2′	0′	2	0	0	0	2	2
2′	1′	3′	2′	2	4	8	2	4	6
2′	2′	0′	0′	2	2	4	0	0	4
2′	3′	1′	2′	2	4	8	2	4	6
3′	0′	3′	0′	4	0	0	0	4	4
3′	1′	0′	3′	4	4	16	4	0	8
3′	2′	1′	2′	4	2	8	2	4	6
3′	3′	2′	2′	4	4	16	2	2	8

From the above table it is clear that for all, we have,

$o(a+b) \leq o(a)+o(b)$
$o(a.b) \leq \max\{o(a),o(b)\}$
$o(a) \geq 0$ and $o(a)=0$ if $a=0$.

Hence, ||.|| is a triangular norm on *G*

Some fuzzy aspects of nearrings:
(Davvaz [4]) Let μ be a fuzzy subset of a nearring N. Then μ is called a fuzzy ideal with thresholds of N, if for all x, y, i ∈ N, the conditions: $\alpha \vee \mu(x + y) \geq \beta \wedge \mu(x) \wedge \mu(y)$, $\alpha \vee \mu(-x) \geq \beta \wedge \mu(x)$, $\alpha \vee \mu(y + x - y) \geq \beta \wedge \mu(x)$, $\alpha \vee \mu(xy) \geq \beta \wedge \mu(x)$, $\alpha \vee \mu(x(y + i) - xy\} \geq \beta \wedge \mu(i)$ (here, we call α as the lower threshold of N and β as the upper threshold of N (here, $\mu(0) \geq \beta$)).

(Kedukodi [14], Kedukodi et.al. [7, 8, 10]) A fuzzy ideal μ of N is called (i) equiprime if $\alpha \vee \mu(a) \vee \mu(x\text{-}y) \geq \beta \wedge \inf_{r \in N} \mu(arx - ary)$, for all x, y, a ∈ N, (ii) 3-prime if for all a, b ∈ N, $\alpha \vee \mu(a) \vee \mu(b) \geq \beta \wedge \inf_{r \in N} \mu(arb)$, and (iii) c-prime $\alpha \vee \mu(a) \vee \mu(b) \geq \beta \wedge \inf_{r \in N} \mu(ab)$. (One can observe that in case of commutative rings, all these concepts coincide). (Bhavanari and Kuncham [92, 95]) For a zero symmetric gamma nearring, the concepts fuzzy ideal, fuzzy prime ideals, and fuzzy cosets are defined.

Gamma Nearrings and a generalization:
Let (M, +) be a group (not necessarily Abelian) and Γ be a non-empty set. Then M is said to be a Gamma nearring if there exists a mapping $M \times \Gamma \times M \to M$ (denote the image of (m_1, α_1, m_2) by $m_1\alpha_1m_2$ for $m_1, m_2 \in M$ and $\alpha_1 \in \Gamma$) satisfying the following conditions: (i) $(m_1 + m_2)\alpha_1m_3 = m_1\alpha_1m_3 + m_2\alpha_1m_3$ and (ii) $(m_1\alpha_1m_2)\alpha_2m_3 = m_1\alpha_1(m_2\alpha_2m_3)$, for all $m_1, m_2, m_3 \in M$ and for all $\alpha_1, \alpha_2 \in \Gamma$. Furthermore, M is said to be a zero-symmetric gamma nearring if $m\alpha 0 = 0$ for all $m \in M$, $\alpha \in \Gamma$ (where '0' is additive identity in M).

Let M be a Γ-nearring. An additive group G is said to be a Γ-nearring-module (or $M\Gamma$-module) if there exists a mapping $M \times \Gamma \times G \to G$ (denote the image of (m, α, g) by $m\alpha g$ for $m \in M$, $\alpha \in \Gamma$, $g \in G$) satisfying the conditions

(i) $(m_1 + m_2)\alpha_1 g = m_1\alpha_1 g + m_2\alpha_1 g$ and
(ii) $(m_1\alpha_1m_2)\alpha_2 g = m_1\alpha_1 (m_2\alpha_2 g)$
for $m_1, m_2 \in M$, $\alpha_1, \alpha_2 \in \Gamma$ and $g \in G$.

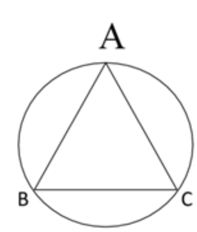
Hamsa et.al [9] we define a generalization of both the structures gamma nearring and $M\Gamma$-module namely $\theta\Gamma$-N-group.

Definition: Let (G, +) be a group. G is called a $\Theta\Gamma$-N-group if there exists a nearring (N, +, ·) and there exist maps $\Theta(N \times \Theta \times G \to G)$, $\Gamma(N \times \Gamma \times N \to N)$ containing nearring multiplication ·, $\Delta_\Gamma(N \times \Delta_\Gamma \times G \to G)$ satisfying the following conditions.

1. θ is right distributive: $(n + m)\theta g = n\theta g +_G m\theta g$, for all $n, m \in N$, $g \in G$, $\theta \in \Theta$;
2. θ is quasi associative: for every $n, m \in N$, $\gamma \in \Gamma$, there exists $\delta_\gamma \in \Delta_\Gamma$ such that $(n\gamma m)\theta g = n\delta_\gamma(m\theta g)$, for all $g \in G$, $\theta \in \Theta$.

Example of Symmetry of Equilateral Triangle

Example 0.1.1. Consider a triangle as given below:



Let R_0 denote no change in the triangle.

R_{120} be rotation of triangle by 120 degree in anti-clock direction.

R_{240} be rotation of triangle by 240 degree in anti-clock direction.

F_A be flip about the vertex A.

F_B be flip about the vertex B.

F_C be flip about the vertex C.

Take G ={$R_0, R_{120}, R_{240}, F_A, F_B, F_C$}

(G,+) is defined as follows,

+	R_0	R_{120}	R_{240}	F_A	F_B	F_C
R_0	R_0	R_{120}	R_{240}	F_A	F_B	F_C
R_{120}	R_{120}	R_{240}	R_0	F_C	F_A	F_B
R_{240}	R_{240}	R_0	R_{120}	F_B	F_C	F_A
F_A	F_A	F_C	F_B	R_0	R_{240}	R_{120}
F_B	F_B	F_A	F_C	R_{120}	R_0	R_{240}
F_C	F_C	F_B	F_A	R_{240}	R_{120}	R_0

Let N={0,1,2,3,4,5} and (N,+) be defined as (Z_6, +).
Take Γ ={α_1,α_2} defined as

α_1	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1
α_2	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Let Θ ={θ_1, θ_2, θ_3} be defined as

θ_1	R_0	R_{120}	R_{240}	F_A	F_B	F_C
0	R_0	R_0	R_0	R_0	R_0	R_0
1	R_0	R_{120}	R_{240}	F_A	F_B	F_C
2	R_0	R_{240}	R_{120}	R_0	R_0	R_0
3	R_0	R_0	R_0	F_A	F_B	F_C
4	R_0	R_{120}	R_{240}	R_0	R_0	R_0
5	R_0	R_{240}	R_{120}	F_A	F_B	F_C

θ_2	R_0	R_{120}	R_{240}	F_A	F_B	F_C
0	R_0	R_0	R_0	R_0	R_0	R_0
1	R_0	R_{120}	R_{240}	R_0	R_0	R_0
2	R_0	R_{240}	R_{120}	R_0	R_0	R_0
3	R_0	R_0	R_0	R_0	R_0	R_0
4	R_0	R_{120}	R_{240}	R_0	R_0	R_0
5	R_0	R_{240}	R_{120}	R_0	R_0	R_0

θ_3	R_0	R_{120}	R_{240}	F_A	F_B	F_C
0	R_0	R_0	R_0	R_0	R_0	R_0
1	R_0	R_0	R_0	F_A	F_B	F_C
2	R_0	R_0	R_0	R_0	R_0	R_0
3	R_0	R_0	R_0	F_A	F_B	F_C
4	R_0	R_0	R_0	R_0	R_0	R_0
5	R_0	R_0	R_0	F_A	F_B	F_C

For every element in Γ we have δ □ Δ such that Δ ={ $\delta_\alpha 1$, $\delta_\alpha 2$} defined as

$\delta_\alpha 1$	R_0	R_{120}	R_{240}	F_A	F_B	F_C
0	R_0	R_0	R_0	R_0	R_0	R_0

1	R_0	R_{120}	R_{240}	F_A	F_B	F_C
2	R_0	R_{240}	R_{120}	R_0	R_0	R_0
3	R_0	R_0	R_0	F_A	F_B	F_C
4	R_0	R_{120}	R_{240}	R_0	R_0	R_0
5	R_0	R_{240}	R_{120}	F_A	F_B	F_C

$\delta_\alpha 2$	R_0	R_{120}	R_{240}	F_A	F_B	F_C
0	R_0	R_0	R_0	R_0	R_0	R_0
1	R_0	R_{120}	R_{240}	F_A	F_B	F_C
2	R_0	R_{240}	R_{120}	R_0	R_0	R_0
3	R_0	R_0	R_0	R_0	R_0	R_0
4	R_0	R_{120}	R_{240}	R_0	R_0	R_0
5	R_0	R_{240}	R_{120}	F_A	F_B	F_C

Then G is $(N, \Gamma, \Theta, \Delta)$- group.

Additional applications:

One component that has seemed new as an application of nearrings is the use of planar and other nearrings to develop designs and codes. Many authors like Gunter Pilz, Clay and Wen.F. Ke. have contributed a quantum research of planar nearrings and related applications. Planar nearrings are useful to develop several Balanced Incomplete Block designs (BIB designs). Efficient error-correcting codes (linear as well as non-linear) are obtained from nearrings. Well-known concepts of geometry like collineation, dilatation, translation, etc,. are handled with the help of nearrings, giving rise to a "dictionary" between geometry and group theory. One of the applications of group theory is in the study of symmetry. There are several results that explain how conservation laws of physical systems arise from their symmetries under various transformations. Non-linear transformations were handled using algorithms involving nearring generators wherein group theoretical algorithms could not be applied. The GAP package SONATA (abbreviated as: Systems of Nearrings and their Applications). The SONATA package provides (i) methods for the construction and analysis of finite nearrings, (ii) approaches for constructing all endomorphisms and all fixed-point-free automorphisms of a given group, (iii) constructing the nearring of polynomial functions of the group, (iv) obtaining the functions to get solvable fixed-point-free automorphism groups on abelian groups, nearfields, planar nearrings, in addition to block designs from those, etc. The link between nearrings and automata, nearrings and dynamical systems can be found in Pilz [22]. For more details about nearrings and applications, one may visit the nearrings webpage http://www.algebra.uni-linz.ac.at/Nearrings/.

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