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Research Article

Constrained optimization of Mars entry phase trajectory using the modified predictor-corrector method

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Abstract

In this paper, a modified predictor-corrector method is explored to get a constrained optimal trajectory for Mars entry phase. The method uses a parameterized bank angle equation, which is given in terms of specific energy and hence enabling a three-dimensional trajectory optimization. The reference trajectory needed for computation of bank angle in each guidance cycle is obtained using the steepest descent method. Variation in bank angle is restricted. Typical entry path constraints, heat rate, dynamic pressure, and load factor were successfully imposed while minimizing the terminal position and velocity errors.

Keywords: Trajectory optimization, predictor-corrector method, numerical method, bank angle control, constrained optimization, Mars entry phase, trajectory control, path constraints, aerodynamic forces, bank reversal, guidance algorithm.

Introduction

Planetary entry guidance involves manoeuvring the spacecraft entering the planet's atmosphere using the aerodynamic forces to reach a desired altitude with the desired velocity while ensuring that the physical constraints of the spacecraft such as dynamic pressure, load factor and heat rate are within allowable limits. A control variable, either the bank angle or angle of attack can be chosen to effectively manoeuvre the spacecraft.

Predictor-corrector being one of the methods widely used for trajectory optimization cannot be used to satisfy these constraints in its baseline form. Thus, in (1) a modified method was developed that can be used for both low and high lifting vehicles. In this paper, the method given in (1) is explored and used to generate a constrained optimal trajectory for Mars entry phase. The method uses a parameterized bank angle equation in terms of specific energy to predict and correct the trajectory. The aim was to minimize terminal position and velocity errors.

Predictor-corrector method for low lifting vehicles is explained in (2), a modified method to deal with heat constraints in (3), for a two-dimensional entry trajectory optimization, the lateral entry guidance in (4), a constrained predictor-corrector method for three-dimensional entry guidance and tracking is explained in (5).

Section II of this paper focuses on the baseline method and how a modified version of it can deal with constraints. In section III, the application of a modified method to Mars entry phase trajectory optimization is discussed, followed by the conclusion in section IV.

Baseline and modified predictor-corrector methods

The predictor-corrector method in its original form can generate an unconstrained optimal trajectory. The baseline algorithm explained in (1) computes the control variable using specific energy in every guidance cycle, whose sign is decided by the bank reversal logic. This control variable is used to propagate the dynamics with respect to the specific

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energy until the terminal condition is met. If not, then the control variable is updated using the Newton-Rapson method for the next iteration.

In the modified predictor-corrector method, the computed control variable is termed as the base variable and a command variable is computed using the reference altitude rate. The termination criteria and the way of determining the sign of the bank angle remain the same.

The following being the three-dimensional entry dynamics,

$$\dot{x}_1 = x_4 \sin(x_5) \tag{1}$$

$$\frac{\dot{x}_2 = x_4 \cos(x_5) \sin(x_6)}{x_1 \cos(x_3)} \tag{2}$$

$$\dot{x}_3 = \frac{x_4 \cos(x_5) \cos(x_6)}{x_1} \tag{3}$$

$$\dot{x}_4 = -D - \frac{\mu \sin(x_5)}{x_1^2} \tag{4}$$

$$\dot{x}_5 = \frac{1}{x_4}\left[L\cos(\sigma) + \left(x_4^2 - \frac{\mu}{x_1}\right)\frac{\cos(x_5)}{x_i}\right] \tag{5}$$

$$\dot{x}_6 = \frac{1}{x_4}\left[\frac{L\sin(\sigma)}{\cos(x_5)} + \frac{x_4^2 \cos(x_5)\sin(x_6)\tan(x_3)}{x_1}\right] \tag{6}$$

Where,

x1 is altitude,

x2 is longitude,

x3 is latitude,

x4 is velocity,

x5 is flight path angle,

x6 is heading angle.

L and D are, lift and drag accelerations.

The constraints being,

Heat rate is given by,

$$\dot{Q} = k_q\sqrt{\rho}v^{3\cdot 15} \tag{7}$$

Dynamic pressure,

$$q = {}^1\!/_2\, \rho v^2 \tag{8}$$

Load factor,

$$n = \sqrt{L^2 + D^2}/g_0 \tag{9}$$

The method explained in (1) deals with normalized equations. However, none of the equations used here was normalized, instead, the computation was done accordingly. To avoid different unwanted combinations of altitude and velocity for the same specific energy value, the dynamics was propagated with respect to time instead of the specific energy.

For Mars entry phase trajectory, the chosen performance index is as given below,

$$J = \frac{1}{2}((r - r_d)^2 + (v - v_\mathrm{d})^2) \tag{10}$$

The termination criteria were that the error in the current heading angle compared to that of the reference should be within a pre-set value. The method requires a nominal trajectory for computation of energy and thus the base bank angle in each cycle. Steepest descent was used to obtain a nominal trajectory, which satisfies the performance index given above. Along with the modified predictor-corrector method, the overall algorithm ensures that the altitude, velocity and cross-range errors are minimized.

Data and simulation results

The initial conditions are as follows:

Radius of Mars = 33,90,000 m,
Surface area of spacecraft = 8.7 m2,
Mass = 800 kg,
Coefficient of lift, cl = 0.3,
Coefficient of drag, cd = 1.54,
Altitude = 125 km,
Longitude = -114.88°,
Latitude = 37.97°,
Velocity = 4,000 m/s,
Flight path angle = -11.7°,
Heading angle = 36.74°.
Desired target conditions are as follows:
Altitude = 12 km
Velocity = 600 m/s

Mission duration is set as 194 sec based on the parametric study carried out in the steepest descent method.

It was observed that the baseline method by itself could significantly reduce the constraint values, which was evident for load factor if it is ensured that the terminal conditions are met.

On the other hand, neither the baseline nor the modified method given in (1) propagates the velocity at each guidance cycle, instead computes it using the energy, which in turn is calculated using velocity and altitude. Thus, requires the velocity and altitude values from the nominal trajectory at each cycle for propagation.

Figs 3(a) to 3(f) show the simulation results for state variables' variation.

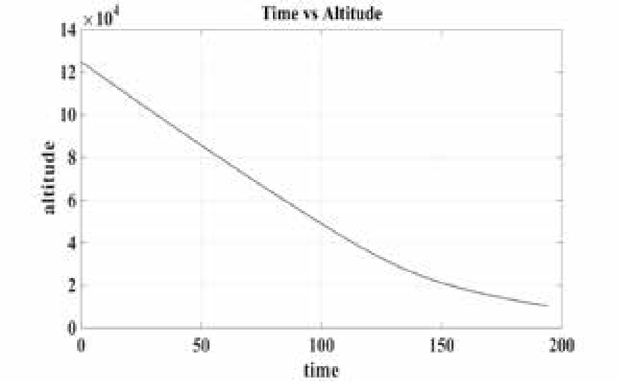

Fig 3(a): Altitude Vs Time

In the above figure, the terminal height of 11.8 km is observed to be achieved.

Figs 3(b) and 3(c) show the variation of longitude and latitude, respectively. Both the parameters are not constrained while defining the performance index.

Fig 3(d) shows that the velocity has converged to nearly 600 m/s.

Flightpath angle, which also is a free parameter, is observed to vary from -12° to -17° while breaking the velocity as shown in Fig 3(e).

Heading angle, which signifies the out of plane movement is observed to vary between 36.5° to 41° leading to the inference that it is significantly out of plane motion. However, the horizontal position is a free parameter since the landing error ellipse will be large in Mars entry descent and landing missions.

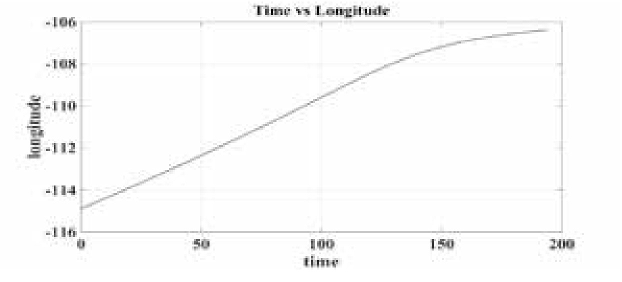

Fig 3(b): Longitude Vs Time

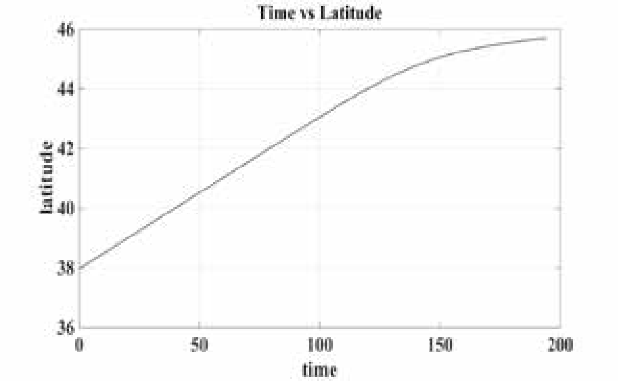

Fig 3(c): Latitude Vs Time

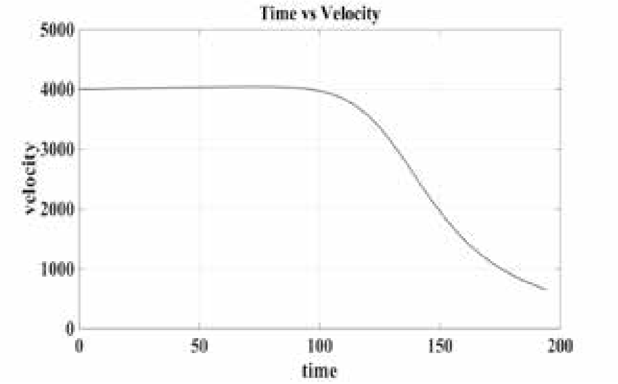

Fig 3(d): Velocity vs Time

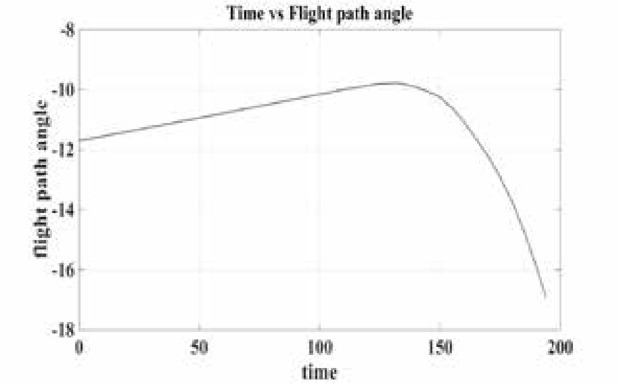

Fig 3(e): Flight Path Angle (degree) Variation

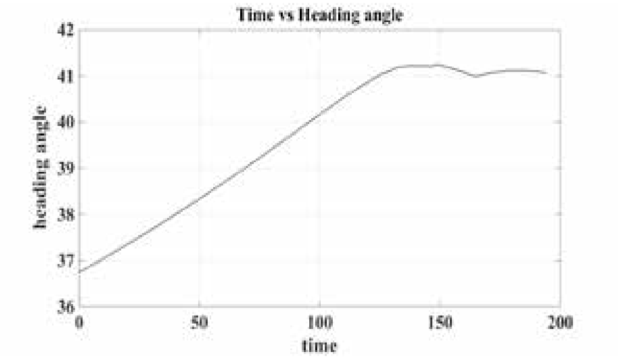

Fig 3(f): Heading Angle (degree)

Fig 3(g) shows the variation of bank angle, which is the control variable. During the first sixty seconds of flight, the bank angle is observed to vary between 0° to +80°. During the next phase, until the end, it is observed to vary between 0° and -80°. This command signal has to be tracked by the Digital Autopilot.

The three entry constraint plots are shown in Figs 3(h) to 3(j), respectively.

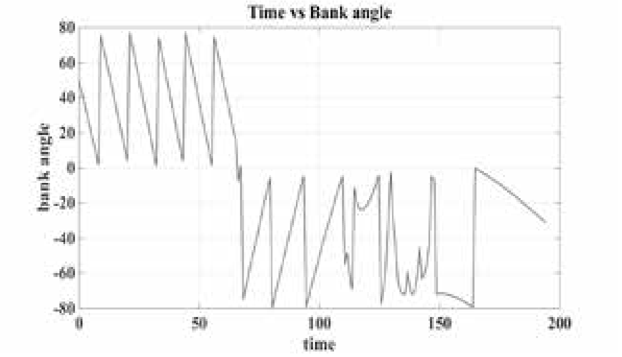

Fig 3(g): Bank angle Vs Time

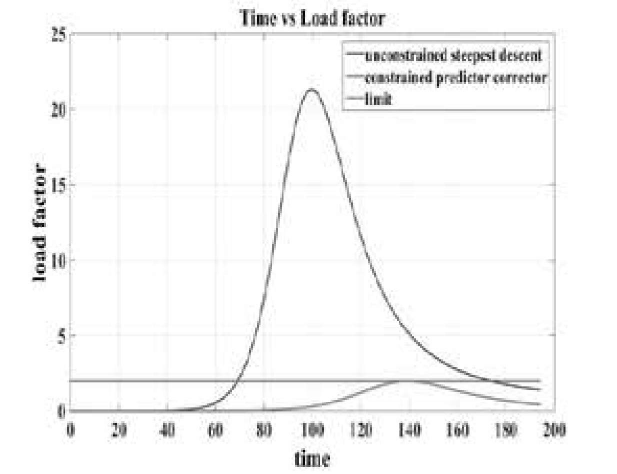

Fig 3(h): Load factor

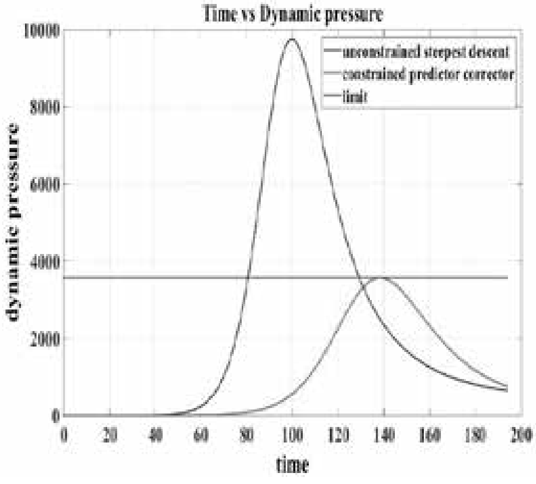

Fig 3(i): Dynamic pressure

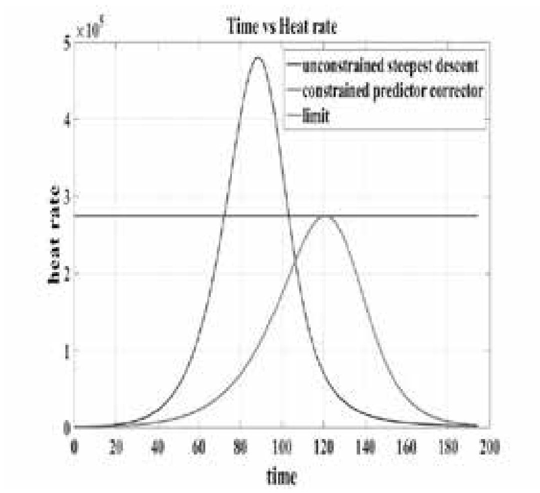

Fig 3(j): Heat rate

Concerning the entry constraints, there is a clear indication that the constraints are getting restrained between the desired limits as compared to unconstrained optimization with Steepest Descent.

Conclusions

The modified predictor-corrector method was explored, and an improved algorithm was applied to Mars entry phase trajectory optimization. Since the original method uses normalized equations and depends on specific energy, a few modifications were made in computation and nominal trajectory was obtained using the steepest descent method. The method used to generate the nominal trajectory would affect the robustness of the algorithm, as it does not use the dynamic equation to propagate the velocity. The rate at which the bank reversal happens can be controlled to a significant extent. Minimum and maximum limit within which the bank angle can vary was set to be -80 to +80 degrees. It was observed that for the same initial and final conditions, the modified predictor-corrector method successfully restrains the constraints.

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