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Hybrid expansion law for accelerating universe in FRW Cosmology

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Research Article

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Abstract

We propose a cosmological model in the form of a hybrid expansion law wherein the scale factor is given by the product of three terms. These terms include a power-law $a(t)\sim t^{B}$, an exponential law $a(t)\sim t^{At}$ and a damping term $a(t)\sim t^{-c/t}$. Equivalently the proposed hybrid law takes the form $H(t) = A + B/t + C/t^2$, where H(t) is Hubble parameter in terms of cosmic time *t*. We confirmed that the model is in agreement with the observational data of Hubble parameter H(z) and the distance modulus μ versus red-shift z data of SNIa Union 2.1. The model also shows the transition from deceleration to acceleration phase at red-shift lying between 0.652 and 0.839.

Keywords: Cosmological model, scale factor, hybrid expansion law, Hubble parameter, SNIa Union 2.1

Introduction

The problem of the evolution of our universe and its ultimate fate has long been the subject of human thought and speculation. The late-time cosmic acceleration discovered in 1998 [1,2] has renewed the interest of cosmologists across the globe. Subsequently, the accelerated expansion of the universe has been confirmed by several independent probes such as WMAP [3], Planck [4], SDSS [5], and others. Dark energy (DE) is one of the most popular explanations of acceleration and it is assumed to be an unknown fluid characterized by equation-of-state (EoS) parameter ω, defined by ω=p/ρ where p and ρ are the pressure and energy density of the fluid. The Friedmann-Robertson-Walker (FRW) metric representing a homogeneous and isotropic universe is the foundational basis of modern Cosmology within the framework of General Relativity. The FRW metric admits a scale factor "a" as a function of time t in the form a=a(t) which encompasses the entire cosmic history in itself. Strictly speaking, the dependence of a on t changes as the universe evolves. At early times, when radiation dominated the energy budget of the universe, $a \propto t^{1/2}$ while during intermediate times, when matter dominated the universe and for which, $a \propto t^{1/2}$. At later times, DE dominated the universe for which, $a \propto e^{\Lambda t}$, where Λ is the cosmological constant. The universe has undergone a transition from decelerated expansion to accelerated expansion and therefore the scale factor a have been accordingly hybridized by several authors [6, 7] so as to incorporate such transitions in its expression besides other features of cosmic expansion. Such modified scale factors are commonly referred to as hybrid expansion law (HEL) [8, 9]. A simple form of HEL includes a product of power-law and exponential type of functions. For instance, $a \propto t^{2/3}e^{At}$. An expansion law can be taken as a cosmological model if it can mimic the expansion history of the universe and if it agrees well with the current observational data. It must be fine-tuned with the present cosmological parameters. A HEL of the form $a \propto t^{B}e^{At}$ have been investigated by several authors [6-8] wherein Goswami et al., [7] have reported that the law is in good agreement with the SNIa Union 2.1 data [10]. In the relation, $a \propto t^{B}e^{At}$, A and B are positive constants. In [6], Akarsu et al., have pointed out that the law is good at describing the radiation-dominated era and the DE acceleration era. However, the HEL does not mimic the matter-dominated era properly. In terms of the Hubble

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parameter, H=ȧ/a, where the over-dot represents differentiation with respect to cosmological time t, the HEL $a \propto t^B e^{At}$ can be equivalently expressed as H=A+B/t.

The cosmic history is also explored by suitably parametrizing the equation of state (EoS) parameter ω. The scale factor can be expressed as $a \sim t^{2/3(1+\omega)}$, for radiation ω=1/3, for matter ω=0, for DE fluid ω<-1/3 and for vacuum energy or cosmological constant, which is a candidate of DE, ω=-1. Most of the observed data indicate that the DE-EoS parameter ω_{de} lies within a narrow strip centered around ω_{de}=-1. The 'energy conditions' of classical general relativity implies that ωde>-1(quintessence) which also guarantees that the entropy of the universe is positive [11]. However, observational data do not rule out the possibility that ω_{de}<-1 (phantom) [3,4,10,12,13]. The possibility of such arena with ω_{de}<-1 may lead to future singularity [14, 15]. Models involving the interaction of dark energy and dark matter (DM) have been widely studied and such models are found to reproduce singular behaviour of phantom-like scenarios [16]. Various models of scale factor have been proposed and investigated which describe transitions in the universe from ω_{de}>-1 to ω_{de}<-1 or from deceleration phase to acceleration phase or from matter-dominated era to DE dominated era. In [17], authors have proposed Hubble parameter in the form H = A + 2/3t + B $(t_s - t)^\alpha$. This kind of model is assumed to incorporate the issues of a matter-dominated era which are not addressed properly in HEL of the form $a \propto t^B e^{At}$ [6] or equivalently in H=A+B/t.

In this paper, we propose a similar kind of cosmological model in the form of a HEL wherein the scale factor is given by the product of three factors and it involves three unknown constants. We constrain the values of these constants in the light of recently observed cosmological parameters, namely the Planck results [4].

Then, we investigate the model with reference to (i) distance modulus μ versus red-shift z data of SNIa Union 2.1 compilation [10] and (ii) measured values of Hubble parameter H(z) data. We also investigate the variation of deceleration parameter q with red-shift and look for the transition from deceleration to acceleration phase of the universe.

FRW spacetime and hybrid expansion law

The FRW metric representing a homogeneous and isotropic universe is described by the line element

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right], \quad (1)$$

where k = +1, 0. -1 for closed, flat or open universes respectively. The field equations for FRW metric are given by the following equations (Friedman equations):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad (2)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \text{ -} \quad (4)$$

The Hubble parameter H=ȧ/a ought to be continuously varying with time as it is determined by the total energy density ρ and the curvature k. Eq. (3) is a form of the law of conservation of energy from which one can determine ρ=ρ(a). In eq. (4) ä=dȧ/dt. The deceleration parameter q is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \quad (5)$$

We consider a flat universe (k=0) as most of the cosmic probes [3, 4] indicate towards zero curvature. Then, the deceleration parameter q can be expressed as

$$q = \frac{\rho+3p}{2\rho}. \quad (6)$$

Expression for the present value of deceleration parameter q_0 can be written as

$$q_0 = \frac{1+3\Omega_{de,0}\omega_{de}}{2}, \quad (7)$$

where $\Omega_{de,0}$ is the present value of the dark energy density parameter. As the matter density is always less than the total energy density, eq. (6) implies that q<1/2. From eq. (7) it is obvious that q_0 is negative because ω_{de}<-1/3 as demanded by eq. (4) for positive acceleration.

In the present work we propose an expression of the Hubble parameter as a function of cosmic time as shown below:

$$H(t) = \frac{\dot{a}}{a} = A + \frac{B}{t} + \frac{C}{t^2}. \quad (8)$$

As mentioned before, a scale factor of the form $a \propto t^B e^{At}$ [7] is equivalent to the Hubble parameter in the form H(t)=A+B/t. The additional term C/t^2

introduced in our present work is expected to take into account the effects of the matter-dominated era which are not accommodated properly in H(t)=A+B/t model. Integration of eq. (8) yields the scale factor and our HEL takes the form

$$a(t) = \left(\frac{t}{t_0}\right)^B exp\{A(t-t_0)\}\, exp\left\{-C\left(\frac{1}{t}-\frac{1}{t_0}\right)\right\}. \quad (9)$$

Here t_0 is the present age of the universe. In eq. (9), the present value of the scale factor has been set to unity, that is, $a(t_0)=1$. The constants A and B must be positive numbers for an expanding universe. The exponential term involving the constant C can be thought to represent damping during the intermediate period of matter-dominated era. So, we expect C to be positive too. Once values of constants A, B, and C are determined, the proposed HEL can be tested against the available observational data. We seek to find these constants by constraining our HEL to the cosmological parameter results from the Planck measurements [4] of the cosmic microwave background anisotropies. We take the following values of the cosmological parameter measured at the present time t_0:

Matter density parameter $\Omega_{m,0}$ = 0.3153±0.0073
Spatial curvature density parameter
$\Omega_{k,0}$ = 0.001±0.002,
Dark energy density parameter
$\Omega_{de,0}$=0.6847±0.0073,
Hubble constant H_0 = 0.023,
Present age of the universe t_0 = 13.797±0.023 Gyr.

With the above parameters and ω_{de}=-1, the present value of the deceleration parameter is given by eq. (7) as q_0= -0.52705. We determine the constants A and C using the following equations:

$$C = (1+q_0)H_0^2\frac{t_0^3}{2} - \frac{Bt_0}{2} \quad (10)$$

$$A = H_0 - \frac{B}{2t_0} - (1+q_0)H_0^2\frac{t_0}{2} \quad (11)$$

We use the above-mentioned cosmological parameters and a logically suitable trial value of B in the above equations. We expect B to be very close to its value obtained from the HEL H(t)=A+B/t while constrained with Planck data. This value comes out to be 0.4085. Also, one can get some clue for the initial value of B from the facts that for the radiation-dominated universe, B=0.5 as $a \propto t^{1/2}$ and for the matter-dominated universe, B=0.6667as $a \propto t^{2/3}$. Next, we fit the model with the available observational data and look for the value of B which gives the minimum chi-square (χ^2).

The determination of cosmological parameters is very crucial as the entire expansion history of the universe depends on these values. The Hubble constant H_0 being the most important one. The measured values from 43 independent missions are found to vary from about 67 km $s^{-1}Mpc^{-1}$ to 77 km $s^{-1}Mpc^{-1}$. Among the recently measured values include H_0= 74.03±1.42 [18] which is significantly different from H_0= 67.4±0.5 [4]. The Hubble parameter H(t) is a decreasing function of cosmic time t. Normally, the Hubble parameter is expressed as a function of red-shift z which is related to scale factor by the equation a(t)= 1/((1+z)) and with time by the relation dz/dt=-H(1+z). The measured values of the Hubble parameter as a function of red-shift z have been presented in Table 1.

Table 1: Measured values of Hubble Parameter H(z) from different observations.

Red-shift z	H(z)	Ref.	Red-shift z	H(z)	Ref.	Red-shift z	H(z)	Ref.
0.0708	69.0 ± 19.68	19	0.4247	87.1 ± 11.2	26	0.875	125.0 ± 17.0	22
0.09	69.0 ± 12.0	20	0.43	86.45 ± 3.68	23	0.88	90.0 ± 40.	29
0.12	68.6 ± 26.2	19	0.44	82.6 ± 7.8	27	0.9	117.0 ± 23.0	21
0.17	83.0 ± 8.0	21	0.4497	92.8 ± 12.9	26	1.037	154.0 ± 20.0	22
0.179	75.0 ± 4.0	22	0.47	89 ± 50	28	1.3	168.0 ± 17.0	21
0.199	75.0 ± 5.0	22	0.4783	80.9 ± 9.0	26	1.363	160.0 ± 33.6	31
0.2	72.9 ± 29.6	19	0.48	97.0 ± 62.0	29	1.43	177.0 ± 18.0	21
0.24	79.69 ± 2.65	23	0.51	90.9024	25	1.53	140.0 ± 14.0	21
0.27	77.0 ± 14.0	21	0.57	92.4 ± 4.5	30	1.75	202.0 ± 40.0	21
0.28	88.8 ± 36.6	19	0.593	104.0 ± 13.0	22	1.965	186.5 ± 50.4	31
0.35	84.4 ± 7.0	24	0.6	87.9 ± 6.1	27	2.34	222.0 ± 7.0	32

Red-shift z	H(z)	Ref.	Red-shift z	H(z)	Ref.	Red-shift z	H(z)	Ref.
0.352	83.0 ± 14.0	22	0.61	98.9647	25	2.36	226.0 ± 8.0	33
0.38	81.2087	25	0.68	92.0 ± 8.0	22			
0.382	83.0 ± 13.5	26	0.73	97.3 ± 7.0	27			
0.4	95 ± 17.0	21	0.781	105.0 ± 12.0	22			

We calculate the Hubble Parameter H(z) from our model and compare it with the currently available H(z) data fromTable1. The outcomes are discussed in the next section. Next, we compare our model with the μ versus z data of SNIa Union 2.1 compilation. The distance modulus μ of a source is defined as

$$\mu = 5log_{10}[c(1+z)]\int_0^z \frac{dz'}{H(z')} + 25. \quad (12)$$

Here c is the speed of light. We solve eq. (12) numerically to obtain μ versus z data following our model and compare the same with the SNIa Union 2.1 observational data. We also estimate the deceleration parameter q on the basis of our model and verify whether the model mimics the transition from deceleration to acceleration phase in the recent past. Numerical estimation of deceleration parameter q will be carried out from the following relation, which has been derived using eq. (5) and (9):

$$q = -1 + \frac{(B/t^2 + 2C/t^3)}{H^2}. \quad (13)$$

Results

Hubble Parameter *H* was calculated numerically over a suitable range of red-shift. First of all, we inserted the Planck values of the cosmological parameter at present time in eqs. (10,11) to determine the values of the constants *A* and *C*. Moreover, as already pointed out in the previous sections, we also need to put in a value of *B* in eqs. (10-11) in order to find *A* and *C*. We chose five different values of B in the neighbourhood of 0.4 for the reason already discussed above. These values are given in Table 2. Using five sets of values of *A, B* and *C* we plotted fives curves of Hubble Parameter H(z)as a function of red-shift z in Figure 1.

Table 2: $\chi^2$of the model for different values of constant B.

Value of *B*	*A*	*C*	x^2 (from 43 H(z) data)	x^2 (from 581 SNIa data)
0.1	0.048932	2.12817	257	873
0.2	0.045308	1.43832	163	708
0.3	0.041684	0.74846	82	669
0.4	0.038060	0.05861	32	705

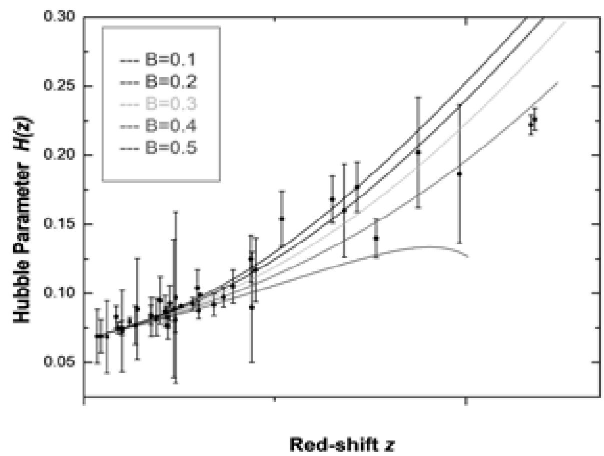

Fig 1: Hubble Parameter at different Red-shift . The scattered dot points in black colour with error bars represent the measured values of . The lines in different colours denote the values given by our model. The five lines from top to down represent the model values for B = 0.1 to 0.5 respectively.

The five lines in different colours from top to down represent model values for *B*= 0.1 to 0.5 respectively. The scattered dots with error bars in black colour represent the measured values of *H(z)*. The lowermost curve representing *B*= 0.5 can be rejected just by visual inspection of Figure 1. We obtained the X^2 of each of the remaining four lines with respect to the 43 measured values of *H(z)* using the relation

$$\chi^2 = \sum_1^{43} \frac{(H_{obs} - H_{HEL})^2}{(\Delta H_{obs})^2}. \quad (14)$$

The χ^2 values for H(z) versus z data were calculated at four different values of B and are depicted in the third column of Table 2. Using the same sets of values of A, B and C, we plotted curves for μ versus z in Figure 2. The overall agreement between our model and SNIa Union 2.1 data is good. Visually, it is difficult to distinguish the four different curves in Figure 2 and it appears that all the curves are in agreement with the SNIa Union 2.1 observational data. The predictions of the model for all the four values of B at lower red-shifts are found to match with each other and they slowly differ over higher red-shifts. The value of distance modulus μ at z = 1.4 for B = 0.1 and that for B = 0.4 differ by 0.4% only.

In order to visualize these differences among these curves and also with reference to the observed data, we plotted another graph for z = 0.8 to z = 1.4 and the curves are depicted in Figure 3. We also obtained the χ^2 values for these curves using the 581 observed μ versus z data. These χ^2 values are presented in the last column of Table 2.

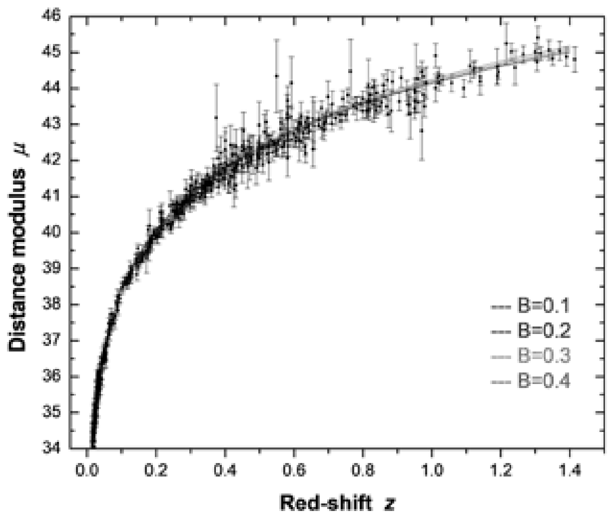

Fig 2: Plot of Distance moduli μ at different Red-shift z. The scattered dot points in black colour with error bars represent the measured values of μ. The lines in different colours denote the values given by our model. The four lines from down to top represent the model values for B = 0.1 to 0.4 respectively.

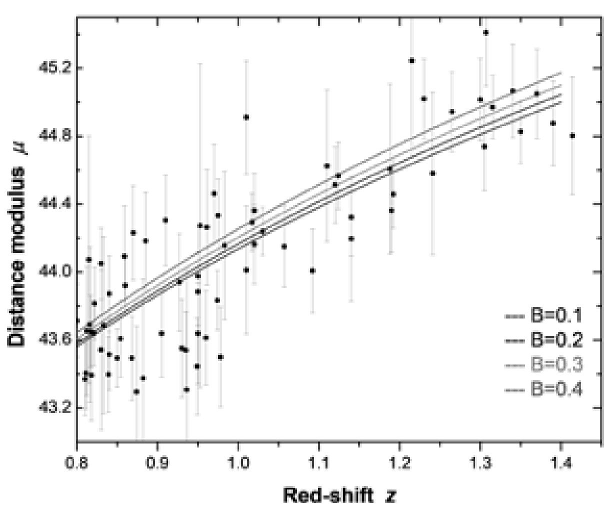

Fig 3: Plot of μ versus z for z = 0.8 to v = 1.4. The scattered dots in black colour with error bars represent the measured values of μ from SNIa Union 2.1 observational data. The lines in different colours denote the values given by our model. The four lines from down to top represent the model values for B = 0.1 to 0.4 respectively.

Our model gives the lowest value of χ^2 = 32 when its prediction at B = 0.4 is compared with the 43 numbers of H(z) observational data. On the other hand, the model's prediction at B = 0.3 yields the lowest χ^2 = 669 when computed with the 581 numbers of SNIa Union 2.1 data. A χ^2 of 32 for 43 H(z) data does not necessarily mean that B = 0.4 is the best value for our model as the observed H(z) data has an average error of 17% and contains error up to 63%. We also obtained χ^2 at an intermediate value of B = 0.365 of for both H(z) data and supernova data and their values are 30 and 695 respectively. Thus, we find that the present model is in good agreement with the available data for B ranging from 0.3 to 0.4.

Finally, the deceleration parameter q was estimated using eq. (13). Our model clearly shows the transition from deceleration to acceleration phase. For B = 0.3, the red-shift near which the transition occurred was found to be z_{tr} = 0.652 and for B = 0.4 it was found to occur near z_tr = 0.839. HELs of [6] and [7] obtained z_{tr} at 0.817 and 0.956, respectively. The curves showing the variations of deceleration parameter q with red-shift z have been shown in Figure 4 at B = 0.3 and B = 0.4.

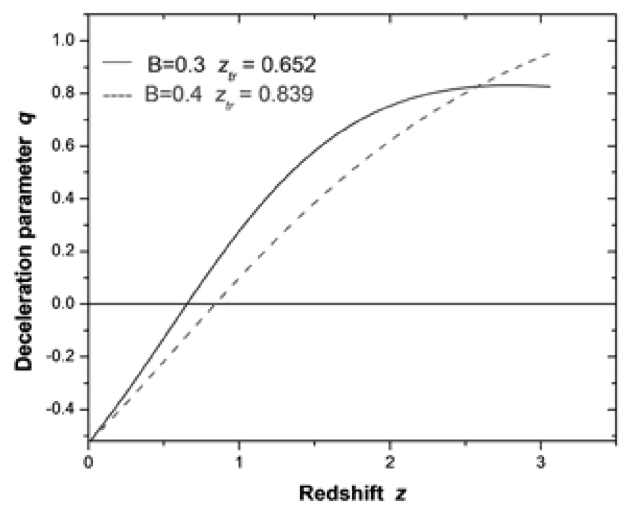

Fig 4: Plots showing the variation of deceleration parameter q with red-shift z. The solid line in blue colour represents q at B = 0.3 while the dashed line in magenta colour represents q at B = 0.4.

Conclusion

We have proposed a hybrid expansion law $a(t) \sim t^B$ incorporating a power-law $a(t) \sim t^{At}$, an exponential law and a damping term $a(t) \sim t^{-c/t}$ which decreases with time. We have obtained the values of constants, and in the framework of the latest Planck results. We also verified that the model is in agreement with the observational data of Hubble Parameter and the distance modulus versus red-shift data of SNIa Union 2.1. Further, we finely tuned the values of constants by finding X^2 for the observed data. We found that the proposed cosmological model in the form of HEL $H(t)=A+B/t+C/t^2$ is in good agreement with SNIa Union 2.1 data for B= 0.3 and with H(z) data for B = 0.4. Furthermore, our model clearly shows the transition from deceleration to acceleration phase of the universe. Our results indicate that the transition could have occurred at a value of red-shift lying between z_{tr} = 0.652 and z_{tr} = 0.839.

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