## Manipal Journal of Science and Technology

Volume 4 | Issue 2 Article 5

12-1-2019

# Counting the number of possible fuzzy controllers

Ambalal Vinayak Patel
*Aeronautical Development Agency, Bangalore, India*, avpatel@nal.res.in

Research Article

# Counting the number of possible fuzzy controllers

**Ambalal Vinayak Patel**

Email: avpatel@nal.res.in

## Abstract

Fuzzy logic is prominent among all types of logics due to the inherent ability to express human intelligence qualitatively. Thus, it is being used in a variety of applications to design embedded intelligent systems. Although it originated to model the complex and ill-defined systems, it eventually also resulted in utilization for designing the intelligent controllers. While designing the fuzzy logic controllers (FLC), we came across a major challenge of selecting the various elements thereof. The flexibility in selecting the combinations of elements leads to arrive at a number of FLCs. Therefore, the challenge came across in selecting the suitable set of FLCs and the best set of FLCs among them for a specific application. This article addresses and explains the way of computing such a number of possible FLCs.

**Keywords**: Analytical structure, controller, fuzzy logic, number, possible

## Introduction

Fuzzy logic, neural networks, genetic algorithms and many such tools are used to express the qualitativeness of the linguistic information/ intelligence and which eventually results in defining and designing the intelligent systems. Fuzzy logic is prominent among all types of logics due to the inherent ability to express human intelligence qualitatively. Thus, it is being used in a variety of applications to design embedded intelligent systems. Although it was originated to model the complex and ill-defined systems, it eventually also resulted in utilization for designing the intelligent controllers.

Deriving the analytical structure of fuzzy controllers is very important as it creates a solid foundation for better understanding, insightful analysis, and more effective designs of fuzzy control systems. A technique was developed for deriving the analytical structure of the fuzzy controllers that Zadeh fuzzy AND operator and the symmetric, identical trapezoidal or triangular input fuzzy set. Many fuzzy controllers use arbitrary trapezoidal/triangular input fuzzy sets that are asymmetric. Extending, therefore, the original technique, a novel method was presented to derive the analytical structure for Mamdani type or TS type rule-based controller, that employs the arbitrary trapezoidal input fuzzy sets and Zadeh fuzzy AND operator. Given the importance of PID control, the focus was made on Mamdani fuzzy PI and PD controllers and was shown in detail how to use the new technique for different configurations of the fuzzy PI/PD controllers. The controllers use two arbitrary trapezoidal fuzzy sets for each input variable, four arbitrary singleton output fuzzy sets, four fuzzy rules, Zadeh fuzzy AND operator, and the centroid defuzzifier. This configuration is more general and complicated than the Mamdani fuzzy PI/PD controllers available at that time in the literature. Therefore, it is called the generalized fuzzy PI/PD controller [6]. Researchers have made efforts to derive the analytical structures of the FLCs with different combinations of the type of fuzzy sets, and fuzzy operators [1-5].

Analytical structure for a fuzzy PID controller is introduced by employing two fuzzy sets for each of the three input variables and four fuzzy sets for the output variable. This structure is derived via left and

*Ambalal Vinayak Patel*

*Integrated Flight Control Systems (IFCS) Directorate, Aeronautical Development Agency (ADA), P B 1718, Vimanapura Post, Bangalore – 560 017, India*

Manuscript received: 3 October 2019
Revision accepted: 28 October 2019

**How to cite this article:** Patel A V. "Counting the number of possible fuzzy controllers", Manipal J. Sci. Tech., vol.4(2), 26-35, 2019.

right trapezoidal membership functions for inputs, trapezoidal membership functions for output, algebraic product triangular norm, bounded sum triangular co-norm, Mamdani's minimum inference method, and centre of sums (COS) defuzzification method. Conditions for bounded-input bounded-output (BIBO) stability are derived using the Small Gain theorem. Two numerical examples along with their simulation results are included to demonstrate the effectiveness of the simplest fuzzy PID controller [7].

It was emphasized that the analytical structure of a fuzzy controller should be investigated in such a way that the structure is sensible in the context of control theory [8]. Also, a brief review of the linear PID, PI, and PD controllers was presented therein as most of the T2 fuzzy controllers. The chapter defines the common components of the T2 fuzzy controllers. It shows various techniques for deriving the explicit analytical structures of four different types of T2 Mamdani fuzzy controllers with two input variables and links the resulting structures to the PI and PD controllers. The chapter provides yet another derivation technique for a class of T2 TSK fuzzy controllers with two input variables and shows how the analytical structure obtained ties to the PI and PD controllers. It also establishes some design guidelines for the T2 Mamdani fuzzy controllers [8].

Analytical structures of the simplest fuzzy PI/PD controllers of Takagi-Sugeno (TS) type are found with a modified rule base [9]. The rule base contains only two rules, which greatly reduce the tuning parameters of the controllers. These controllers are the simplest since minimal number (two) of fuzzy sets are employed on each input variable. The triangular norms and co-norms are chosen as algebraic product/minimum and bounded sum/maximum, respectively. It is shown that the fuzzy controller with modified TS rule base is equivalent to a nonlinear variable gain PI/PD controller. The gain variations and computational aspects are also studied [9].

A new mathematical model of the simplest fuzzy PID controller was revealed which employs two fuzzy sets (negative and positive) on each of the three input variables (error, change in error and double change in error) and four fuzzy sets on the output variable (incremental control) [10]. Different types of membership functions are considered in the fuzzification process of input and output variables. Controller modelling is done via algebraic product AND operator, maximum OR operator, and height (Ht) defuzzification. Detailed analysis of the resulting nonlinear model is presented [10].

This paper proposes the equivalence between fuzzy proportional-integral-derivative (PID) controllers and conventional PID controllers. A well-designed conventional PID controller, with the help of the proposed method, can be rapidly transformed to an equivalent FLC by observing and defining the operating ranges of the input/output of the controller. Furthermore, the knowledge base of the proposed equivalent fuzzy PID controller is represented as a cube fuzzy associative memory (FAM), instead of a combination of PD-type and PI-type FLCs in most research. Simulation results show the feasibility of the proposed technique, both in continuous and discrete-time. Since the design techniques of conventional linear PID controllers have matured, they can act as preliminary expert knowledge for nonlinear FLCs designs. Based on the proposed equivalence relationship, the designer can further tune the membership functions of fuzzy variables in the control rules to exhibit the nonlinearity of an FLC and yield more satisfactory system responses efficiently [11].

While designing the FLCs, we came across facing the challenge of selecting the various elements thereof as listed below:

1. Number of input variables and their definitions or computational model, i.e., error, rate of error, etc.,
2. Number of fuzzy sets in each input variables,
3. Type of fuzzy sets in each input variable,
4. Arrangement of the fuzzy set for each input variable (overlap span),
5. Use of specific triangular norm (T-norm) to evaluate the antecedent part of all rules,
6. Number of output variables,
7. Number of fuzzy sets in each output variables,
8. Type of fuzzy sets in each output variable,
9. Use of specific triangular conorm (T-conorm) to evaluate the outcome of the group of rules of which consequent part has the same output fuzzy set mapped,

10. Type of the fuzzy inference engine (fuzzy reasoning method) used to evaluate the outcome/overall weighting of each rule on the output fuzzy set,
11. Type of defuzzification method used to have a single crisp value by aggregating the outcome of all fired rules.

The flexibility in selecting the combinations of elements results in having several FLCs. Therefore, challenge come across is in selecting the suitable set of FLCs and the best set of FLCs among them for the specific application. This article addresses and explains the way of computing such a number of possible FLCs.

As can be seen from the prior discussion, several of the authors in the past have designed the FLCs by considering different elements of the fuzzy logic while retaining the same rule of mapping and the input-output variables, especially for a class of PI/PD/PID controllers. Derivation of analytical structures of such fuzzy two/three-term controllers and their simulation studies have been demonstrated by considering the different elements of fuzzy controllers involving:

- Fuzzy operators: Combinations of the type of membership functions used for input and output fuzzification, T-norm, T-conorm, fuzzy reasoning/fuzzy inference, and defuzzification methods, and
- Rule base: A fixed set of rules/the mapping between inputs to output fuzzy sets is fixed.

It is felt that such studies stand as a corollary to the set of all such possible FLCs, wherein the same rule of mapping is considered.

Therefore, it is strongly felt that further research is required to arrive at the optimal set of input-output fuzzy mapping or formulation of the rule base. The article also attempts to provide insight to the beginners in this area about the structure of the FLCs and interdependency of the elements thereof in arriving at such a large number of controllers.

The organisation of the paper is given here: After the introduction, Section 2 presents a brief about the generic architecture of the fuzzy controller. Section 3 presents the details on the counting of the number of possible fuzzy controller, first with the input to output mapping with the number of inputs, outputs, and fuzzy sets thereon. Later, the computations for the possible number of fuzzy controllers with the combinations of elements of fuzzy logic, i.e., T-norm, T-conorm, fuzzy reasoning/fuzzy inference, and defuzzification methods are also shown in this Section. Section 4 demonstrates on counting the number of possible fuzzy controllers for the number of inputs ranging from one to four. Section 5 concludes the article.

## Brief about the architecture of the fuzzy controller

The basic architecture of the fuzzy controllers is shown in Fig 1. The type of fuzzification done with labelling the fuzzy sets is illustrated in Fig 2. Fig 3 shows the details of the simplest fuzzy PI controller [2], i.e., fuzzification on the input variables - velocity (rate of error) and error variables with two fuzzy sets, while three fuzzy sets on the output (incremental output) variable, and the corresponding rule mapping (four rules).

The details of the various T-norms, T-conorms, defuzzification methods, etc. can be found in the references listed in this article as well as cross-references cited therein.

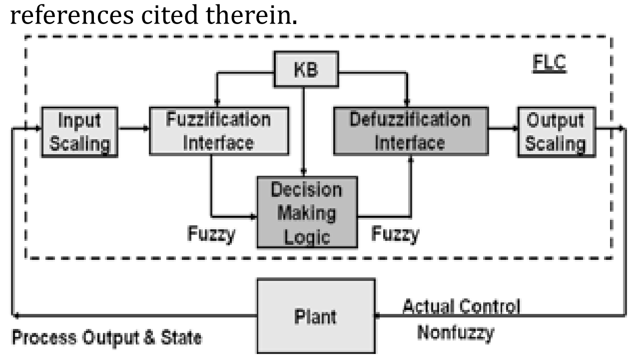


**Fig 1:** Basic configuration of fuzzy logic controller (FLC)

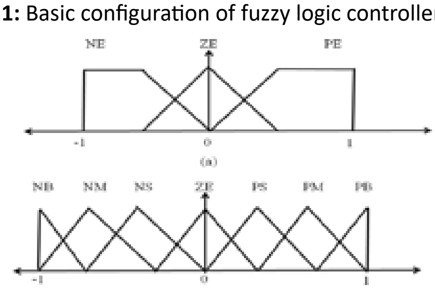


**Fig 2:** Fuzzification on the inputs and/or outputs: (a) Three fuzzy sets: NE-Negative Error, ZE-Zero Error, PE-Positive Error, (b) Seven fuzzy sets with labels: NB-Negative Big, NM-Negative Medium, NS-Negative Small, ZE-Zero, PS-Positive Small, PM-Positive Medium, and PB-Positive Big.


Published by Impressions@MAHE, 2019

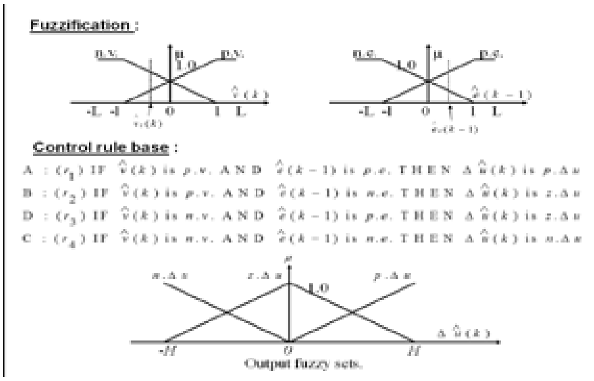


**Fig 3:** Fuzzy PI controller [2]:

- Fuzzification of Velocity (rate of error) and Error inputs with two fuzzy sets,
- Rule base R1 to R4, and
- Fuzzification of output (incremental output) with three fuzzy sets

## Counting the number of possible fuzzy controllers

The computations are shown for the output feedback system with all the following possible combinations of single and multiple numbers of inputs and outputs:

1. Single input and single output (SISO),
2. Single input and multiple outputs (SIMO),
3. Multiple inputs and single output (MISO), and
4. Multiple inputs and multiple outputs (MIMO)

The following nomenclatures have been used in this article:

UOD = Universe of discourse,

$N_I$ = Number of input variables,

$N_O$ = Number of output variables,

$N_{IF}$ = Number of fuzzy sets in the UOD of the input variable,

$N_{OF}$ = Number of fuzzy sets in the UOD of the output variable,

$N_{IF\ iNI}$ = Number of fuzzy sets in the UOD of iNIth input variable,

$N_{OF\ iNO}$ = Number of fuzzy sets in the UOD of $i_{NO}^{th}$ output variable,

$i_{NI}$ = Index for counting the number of inputs ($i_{NI}$ = 1,2,3, …, $N_I$),

$i_{NO}$ = Index for counting the number of outputs ($i_{NO}$ = 1,2,3, …, $N_O$),

R = Number of all possible rules,

$i_R$ = Index for counting the number of rules ($i_R$ = 1,2,3, …, R),

$N_{CIOM}$ = Number of all possible fuzzy controllers by inputs to outputs rule mapping (with a one fixed combination of the fuzzy operators, i.e., type of memberships for inputs and outputs (Fuzzification), T-norm, T-conorm, fuzzy inference, and defuzzification method),

$i_{NCIOM}$ = Index for counting the number of controllers ($i_{NCIOM}$ = 1,2,3, …, $N_{CIOM}$),

Usually, R>= $N_{OF}$, i.e., the number of all possible rules are more than or at least the same as the number of fuzzy sets in the output variable, otherwise, completeness characteristic of mapping between the input to output shall not be satisfied.

For a one fixed combination of a T-norm and T-conorm, the possible numbers of controllers that can be obtained are shown below:

The total number of possible rules can be,

$$R = N_{IF\_1} \times N_{IF\_2} \times \cdots \times N_{IF\_N_I} = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} \qquad (1)$$

Note that the symbol $\prod$ stands for the product of all the elements indicated by ü ü and varying for $i_{NI} = 1$ to $N_I$.

If the number of fuzzy sets is the same in all inputs variables, i.e.,

$$N_{IF\_1} = N_{IF\_2} = \cdots = N_{IF\_N_I}$$

then

$$R = \left(N_{IF}\right)^{N_I} = N_{IF}^{N_I} \qquad (1a)$$

Considering the completeness of the input to output mapping, i.e., all rules are mapped to output fuzzy sets and thus there is no empty/null mapping left, the total number of possible such rule mappings or the controllers (NCIOM) are:

$$\mathrm{N_{CIOM}} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) \qquad (2)$$

**MIMO system:** Equations (1) and (2) are applicable for the MIMO system, which can be re-written as given below with the assumption that the outputs are correlated with AND operation:

$$R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} \tag{3}$$

$$N_{CIOM}^{MIMO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) \tag{4}$$

**MISO system:** Substituting No=1, and thus the index $i_{NO}$=1 in Equations (1) and (2), the resulting set of all possible number of fuzzy controllers shall be:

$$R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} \tag{3}$$

$$N_{CIOM}^{MISO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_{NO}=1}^{1}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_R=1}^{R} N_{OF\_i_{NO}} \tag{4}$$

**SIMO system:** Substituting NI=1, and thus the index $i_{NI}$=1 in Equations (1) and (2), the resulting set of all possible number of fuzzy controllers shall be:

$$R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} = N_{IF\_1} \tag{5}$$

$$N_{CIOM}^{SIMO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) \tag{6}$$

**SISO system:** Substituting NI=1, NO=1, and thus the index $i_{NI}$=$i_{NO}$=1 in Equations (1) and (2), the resulting set of all possible number of fuzzy controllers shall be:

$$R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} = N_{IF\_1} \tag{7}$$

$$N_{CIOM}^{SISO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_{NO}=1}^{1}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_R=1}^{R} N_{OF\_i_{NO}} \tag{8}$$

The overall collation of the number of possible fuzzy controllers for the above-considered systems is given in Table 1.

Table 1: Number of possible rules and possible set of fuzzy controllers for input to output mapping with a one fixed combination of membership functions, T-norm, T-conorm, inference, and defuzzification method for a MIMO system and subsystems.

| System Inputs ↓\Outputs → | | Single | Multiple |
|---|---|---|---|
| Single | Number of rules | $R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} = N_{IF\_1}$ | $R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} = N_{IF\_1}$ |
| | Set of controllers | $N_{CIOM}^{SISO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_{NO}=1}^{1}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_R=1}^{R} N_{OF\_i_{NO}}$ | $N_{CIOM}^{SIMO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right)$ |
| Multiple | Number of rules | $R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}}$ | $R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}}$ |
| | Set of controllers | $N_{CIOM}^{MISO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_{NO}=1}^{1}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right) = \prod_{i_R=1}^{R} N_{OF\_i_{NO}}$ | $N_{CIOM}^{MIMO} = \prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right)$ |

Notes for Table 1:
- Assumed that outputs are correlated with AND operation for the multi-output system.

Possible number of controllers using combinations of various types of memberships, T-norm, T-conorm, inference method (fuzzy reasoning), and defuzzification methods.

With the use of combinations of different types of membership functions used for input and output fuzzification, T-norm, T-conorm, inference method, and defuzzification methods, the output of the controller is affected and thus it leads to having additional sets of fuzzy controllers.

Let

- $N_{IM}$ = Number` of type of membership functions on the inputs
- $i_{im}$ = Index for counting the number of types of membership functions on inputs ($i_{im}$ = 1,2,3, ..., $N_{IM}$)
- $N_{OM}$ = Number of type of membership functions on the outputs
- $i_{OM}$ = Index for counting the number of types of membership functions on outputs (iom = 1,2,3, ..., $N_{OM}$)

$N_{TN}$ = Number of T-norm
$i_{tn}$ = Index for counting the number of T-norm ($i_{tn}$ = 1,2,3, …, $N_{TN}$)
$N_{TCN}$ = Number of T-conorm
itcn = Index for counting the number of T-conorm (itcn = 1,2,3, …, $N_{TCN}$)
$N_{FR}$ = Number of fuzzy reasoning or inference methods
$i_{fr}$ = Index for counting the number of fuzzy reasoning (ifr = 1,2,3, …, $N_{FR}$)
$N_{DFZ}$ = Number of defuzzification method
$i_{dfz}$ = Index for counting the number of defuzzification method (idfz = 1,2,3, …, $N_{DFZ}$)
$N_{CFOP}$ = Number of possible fuzzy controllers with the combinations of fuzzy operators alone (Type of memberships for inputs and outputs, T-norm, T-conorm, inference methods, and defuzzification) for a one fixed set of rules or input to output mapping
$N_{FC}$ = Total number of possible fuzzy controllers

then

$$N_{CFOP} = N_{IM} * N_{OM} * N_{TN} * N_{TC} * N_{FR} * N_{DFZ} \quad (9)$$

For simplicity and convenience in writing the computations in the subsequent discussions, let

$$N_{CFOP} = N_{CFOP1} * N_{CFOP2} \quad (9a)$$

where,

$$N_{CFOP1} = N_{IM} * N_{OM} \quad (9b)$$

$$N_{CFOP2} = N_{TN} * N_{TC} * N_{FR} * N_{DFZ} \quad (9c)$$

Thus,

$N_{CFOP1}$ = Number of possible fuzzy controllers with the combinations of fuzzy operators (consisting of the type of membership functions for inputs and outputs) for a one fixed set of rules or input to output mapping

$N_{CFOP2}$ = Number of possible fuzzy controllers with the combinations of fuzzy operators (consisting of combinations of T-norm, T-conorm, inference methods, and defuzzification) for a one fixed set of rules or input to output mapping

Therefore, the total number of all possible fuzzy controllers with the combinations of input to output mapping and combinations of different fuzzy operators are:

$$N_{FC} = N_{CIOM} * N_{CFOP} \quad (10)$$

$$N_{FC} = N_{CIOM} * N_{CFOP1} * N_{CFOP2} \quad (10a)$$

Or

$$N_{FC} = N_{CIOM} * (N_{IM} * O_M) * (N_{TN} * N_{TC} * N_{FR} * N_{DFZ}) \quad (10b)$$

where,

$$N_{CIOM} = \prod_{i_{NO}=1}^{N_O} \left( \prod_{i_R=1}^{R} N_{OF\_i_{NO}} \right) \quad (2)$$

$$R = \prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}} \quad (3)$$

The complete overview of the number of possible rules and fuzzy controllers with different combinations of the number of inputs and outputs, a minimal number of fuzzy sets thereon and fuzzy operators are given in Table 2. The computed values of the number of possible controllers for the number of inputs ranging from one to four are also presented in Table 2.

### Aggregated/defuzzified output computation

The defuzzified output is computed as given below:

where,

$$y = \frac{\sum_{i_R=1}^{R} A_{i_R} \times y_{i_R}}{\sum_{i_R=1}^{R} A_{i_R}} \quad (11)$$

y = Defuzzified or aggregated crisp output of the fuzzy controller

$y_{iR}$ = Centroid of the output fuzzy set mapped corresponding to the $iR^{th}$ rule

$A_{iR}$ = Area or weight of the output fuzzy set mapped corresponding to the $iR^{th}$ rule after fuzzy inference.

It may be noted that AiR is a function of the weight or outcome μ computed after

- T-norm operation for the rules having a unique fuzzy set mapped from the output, or
- T-norm and then T-conorm operation for the multiple rules having the same fuzzy set mapped from the output.

However, wherever the rules have been mapped to the same output fuzzy sets, their weights (obtained after T-norm operation) are evaluated by using the T-conorm. The T-conorm operations result in having a unified single weight to all those rules. Thus, during the aggregation /while computing the defuzzified output, the index need not be from iR=1 to R, but it

Patel A V: Counting the number of possible fuzzy controllers

Table 2: Overall view of the number of possible rules and fuzzy controllers with different combinations of the number of inputs and outputs, a minimal number of fuzzy sets thereon, and fuzzy operators

| | | About Inputs | | | About Outputs | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sl. No. | No. of input variables | No. of fuzzy sets on each input variable | No. of all possible rules | No. of output variables | No. of fuzzy sets on output variable | No. of possible fuzzy controllers with input and output mapping only | No. of type of input memberships | No. of type of output memberships | No. of T-norm | No. of T-conorm | No. of fuzzy inference methods | No. of defuzzification methods | No. of possible controllers with the No. of combinations of fuzzy operators alone | Total No. of possible fuzzy controllers |
| Symbol→ | | $N_I$ | $N_{IF\_iNI}$ | R | $N_O$ | $N_{OF\_iNO}$ | $N_{CIOM}$ | $N_{IM}$ | $N_{OM}$ | $N_{TN}$ | $N_{TC}$ | $N_{FR}$ | $N_{DFZ}$ | $N_{CFOP}$ | $N_{FC}$ |
| Symbolic computations → | | | | $\prod_{i_{NI}=1}^{N_I} N_{IF\_i_{NI}}$ | | | $\prod_{i_{NO}=1}^{N_O}\left(\prod_{i_R=1}^{R} N_{OF\_i_{NO}}\right)$ | | | | | | | $N_{IM}*N_{OM}*N_{TN}*N_{TC}*N_{FR}*N_{DFZ}$ | $N_{CIOM}*N_{CFOP}$ |
| Index for counting → | | $i_{NI}=1,2,\ldots,N_I$ | $i_{NIF\_iNI}=1,2,\ldots,N_{IF\_iNI}$ | $i_R=1,2,\ldots,R$ | $i_{NO}=1,2,\ldots,N_O$ | $i_{NOF\_iNO}=1,2,\ldots,N_{OF\_iNO}$ | $i_{NCIOM}=1,2,\ldots,N_{CIOM}$ | $i_{im}=1,2,3,\ldots,N_{IM}$ | $i_{im}=1,2,3,\ldots,N_{IM}$ | $i_{tn}=1,2,\ldots,N_{TN}$ | $i_{tcn}=1,2,\ldots,N_{TCN}$ | $i_{fr}=1,2,\ldots,N_{FR}$ | $i_{dfz}=1,2,\ldots,N_{DFZ}$ | $i_{NCFOP}=1,2,\ldots,N_{CFOP}$ | $i_{NFC}=1,2,\ldots,N_{FC}$ |
| Symbol for applicable note given below the Table → | | | @ | | | @ | # | | | | | | | $ | % |
| No. of possible fuzzy controllers computed using the actual values of the associated elements | 1 | 1 | 2 | 2 | 1 | 2 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
| | 2 | 2 | 2 | 4 | 1 | 3 | 81 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 81 |
| | 3 | 2 | 2 | 4 | 1 | 4 | 256 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 256 |
| | 4 | 3 | 2 | 8 | 1 | 4 | 65536 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 65536 |
| | 5 | 3 | 2 | 8 | 1 | 5 | 390625 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 390625 |
| | 6 | 4 | 2 | 16 | 1 | 3 | 43046721 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 43046721 |
| | 7 | 1 | 2 | 2 | 1 | 2 | 4 | 1 | 1 | 2 | 2 | 1 | 2 | 8 | 32 |
| | 8 | 2 | 2 | 4 | 1 | 3 | 81 | 1 | 1 | 2 | 2 | 1 | 2 | 8 | 648 |
| | 9 | 2 | 2 | 4 | 1 | 4 | 256 | 1 | 1 | 2 | 2 | 1 | 2 | 8 | 2048 |
| | 10 | 3 | 2 | 8 | 1 | 4 | 65536 | 1 | 1 | 2 | 2 | 1 | 2 | 8 | 524288 |
| | 11 | 3 | 2 | 8 | 1 | 5 | 390625 | 1 | 1 | 2 | 2 | 1 | 2 | 8 | 3125000 |
| | 12 | 4 | 2 | 16 | 1 | 3 | 43046721 | 1 | 1 | 2 | 2 | 1 | 2 | 8 | 344373768 |

**Notes for Table 2:**

@ It is considered that all fuzzy sets overlap, i.e., there is continuity in defining the fuzziness on the linguistic variables. There is no restriction on the type of fuzzy sets (triangular, trapezoidal, bell curve, etc.) considered on the inputs and outputs.

\# With one fixed combination of membership functions same for all inputs and outputs, T-norm, T-conorm, inference, and defuzzification method

$ While keeping the fixed input and output mapping

% Assumed that outputs are correlated with AND operation

\- **General notes:**

1) The index for counting has been standardized by adopting the convention as $i_{Nx} = 1, 2, \ldots, N_X$. The 'i' stands for the index, 'N' stands for the total count, and 'x' stands as a suffix for the specific element for which the count is being made.
2) PD/PI/PID controllers are the two or three terms controllers and still the backbone of the industrial control systems despite the advancement in the control theory and implementation thereof. Hence, for illustration, up to three inputs, the computations for the number of the possible fuzzy controller are shown. With the addition of one more input (i.e., four inputs), also computations are shown to have an idea on how the number of possible fuzzy controllers grows drastically.
3) Against the No. of possible fuzzy controllers computed using the actual values of the associated elements; Sl. Nos. 1 to 6 are repeated against Sl. Nos. 7 to 12 with the two numbers of T-norm, T-conorm, and defuzzification methods. It can be seen that with a few additional combinations of fuzzy operators, the number of possible fuzzy controller increases drastically (Compare the numbers in the last column for Sl. Nos. 1 and 7, 2 and 8, 3 and 9 and so on).

**Table 3:** Number of possible fuzzy controllers with rule mapping to output variable (Two inputs and one output simplest fuzzy controller)

| C → | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R1 | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N | N |
| R2 | N | N | N | N | N | N | N | N | N | Z | Z | Z | Z | Z | Z | Z | Z | Z | P | P | P | P | P | P | P | P | P |
| R3 | N | N | N | Z | Z | Z | P | P | P | N | N | N | Z | Z | Z | P | P | P | N | N | N | Z | Z | Z | P | P | P |
| R4 | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P |
| C → | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 |
| R1 | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z | Z |
| R2 | N | N | N | N | N | N | N | N | N | Z | Z | Z | Z | Z | Z | Z | Z | Z | P | P | P | P | P | P | P | P | P |
| R3 | N | N | N | Z | Z | Z | P | P | P | N | N | N | Z | Z | Z | P | P | P | N | N | N | Z | Z | Z | P | P | P |
| R4 | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P |
| C → | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 |
| R1 | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P | P |
| R2 | N | N | N | N | N | N | N | N | N | Z | Z | Z | Z | Z | Z | Z | Z | Z | P | P | P | P | P | P | P | P | P |
| R3 | N | N | N | Z | Z | Z | P | P | P | N | N | N | Z | Z | Z | P | P | P | N | N | N | Z | Z | Z | P | P | P |
| R4 | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P | N | Z | P |

**Notes for Table 3:**

- Four number of rules for two variables with two fuzzy sets in each.
- Number of Mapping/Controllers ($N_{CIOM}$ = C = 81) for the rules to the three fuzzy sets (N: Negative, Z: Zero, and P: Positive) on a single output variable.
- The PD controller rule mapping of Reference [1] corresponds to the controller No. 35 in Table 3 assuming the order of the rules R1 to R4 is same. The input variables are the velocity of error (rate of error) and acceleration (rate of error).
- The PI controller rule mapping of Reference [2] corresponds to the controller No. 15 in Table 3 assuming the order of the rules R1 to R4 is same. The input variables are the velocity of error (rate of error) and error.
- Several authors have used these mappings of the simplest PD and PI controllers with different fuzzy operators to derive the analytical structures [5, 6]. Further, with multifuzzy sets, the same rule mapping has been extended for fuzzy PI and PD controllers [3-4].

**Table 4:** Legends/Colour code/Conventions used in Table 3.

| Colour code | Controller No. | Description |
|---|---|---|
| | 15, 16, 22, 26, 30, 35, 47, 52, 56, 60, 66, 67 | All output fuzzy sets are mapped to the rules and thus they satisfy the completeness criterion. Further, the mapping is symmetrical and thus these mappings could be the suitable candidates |
| | 6, 8, 12, 17, 20, 23, 24, 33, 34, 36, 39, 43, 46, 49, 58, 59, 62, 64, 65, 70, 74 | All output fuzzy sets are mapped to the rules and thus they satisfy the completeness criterion. However, mapping is not in a symmetric manner and thus they may not be the suitable candidates |
| | 2, 3, 4, 5, 7, 9, 10, 11, 13, 14, 18, 19, 21, 25, 27, 28, 29, 31, 32, 37, 38, 40, 42, 44, 45, 48, 50, 51, 53, 54, 55, 57, 61, 63, 68, 69, 71, 72, 73, 75, 76, 77, 78, 79, 80 | Only two output fuzzy sets (out of three) mapped to all rules and thus does not satisfy completeness criterion. Since completeness criterion is not satisfied, it does not matter whether the mapping is symmetric or asymmetric. |
| | 1, 41, 81 | Only one output fuzzy set mapped to all rules and thus does not satisfy the completeness criterion. |

shall be less than R. The reduction is to the only one rule instead of the number of rules aggregated with T-conorm. Let us,

RU = Number of rules having a unique output set mapped.

$R_{TC}$ = Number of rules having the same output fuzzy set. There could be a group of such a number of rules wherein all rules from one group are mapped to a unique output fuzzy set. However, each group has a different output fuzzy set mapped.

$G_{TC}$ = Total number of groups of rules (each group has a unique set mapped, but across the group, different output fuzzy sets are mapped). After the T-conorm operation, each group represents one rule instead of all rules of that group having the same output fuzzy set mapped.

iRU = Index for counting the number of rules having unique output fuzzy sets mapped,

iRTC = Index for counting the number of rules having the same output fuzzy set

iGtc = Index for counting the total number of groups of RTC (iGtc = 1, 2, …, GTC).

$R_{TC\_iGtc}$ = ith group (specifically, iGtc) showing the total number of rules (RTC) in that group

Then, the total number of rules can be expressed as

$$R = R_U + R_{TC} = R_U + \sum_{i_{Gtc}=1}^{G_{tc}} R_{TC\_iGtc} \tag{12}$$

Then the defuzzified output is computed as given below:

$$y = \frac{\sum_{i_R=1}^{R} A_{i_R} \times y_{i_R}}{\sum_{i_R=1}^{R} A_{i_R}} = \frac{\left(\sum_{i_{RU}=1}^{R_U} A_{i_{RU}} \times y_{i_{RU}}\right) + \left(\sum_{i_{Gtc}=1}^{G_{TC}} A_{i_{Gtc}} \times y_{i_{Gtc}}\right)}{\left(\sum_{i_{RU}=1}^{R_U} A_{i_{RU}}\right) + \left(\sum_{i_{Gtc}=1}^{G_{TC}} A_{i_{Gtc}}\right)} \tag{13}$$

where,

$A_{iGtc}$ = Area or weight of the output fuzzy set mapped to the group of rules having the same fuzzy set.

It may be noted that the number of possible fuzzy controllers does not reduce by doing the T-conorm operations; however, it does help in the reduction of the number of computations during the defuzzification process.

**Demonstration of formulation of the possible number of controllers for the two inputs – single output (TISO) system: A subset of the MISO system**

The following values are considered for TISO fuzzy controller with a one fixed set of membership functions for inputs and output, T-norm, T-conorm, inference, and defuzzification method:

$N_{IF\_iNI}$ = 2 = Number of fuzzy sets in the UOD of iNIth input variable (each variable),

$N_{OF\_iNO}$ = 3 = Number of fuzzy sets in the UOD of iNOth output variable,

$N_I$ = 2 = Number of input variables,

$N_O$ = 1 = Number of output variables,

R = 4 = Number of all possible rules,

iNI = 1 to 2 = Index for counting the number of inputs (iNI = 1,2,3, …, $N_I$),

$i_{NO}$ = 1 = Index for counting the number of outputs (iNO = 1,2,3, …, No),

$i_R$ = 1 to 4 = Index for counting the number of rules (iR = 1,2,3, …, R),

NCIOM = C = 81 = Total Number of all possible fuzzy controllers,

$i_{NCIOM}$ = 1 to 81 = Index for counting the number of controllers ($i_{NCIOM}$ = 1,2,3, …, $N_{CIOM}$).

Table 3 presents the number of possible fuzzy controllers with rule mapping to an output variable (Two inputs and one output simplest fuzzy controller). Table 4 presents the legends used in Table 3 and the explanations thereon.

## Conclusions

Fuzzy logic is prominent among all types of logics due to the inherent ability to express human intelligence qualitatively. Thus, it is being used in a variety of applications to design embedded intelligent systems. Although it originated with to model the complex and ill-defined systems, it eventually also resulted in utilization for designing the intelligent controllers. While designing the FLCs, we came across the challenge of selecting the various elements thereof. The flexibility in selecting the combinations of elements results in several FLCs. Therefore, the challenge came across is in selecting the suitable set of FLCs and the best set of FLCs among them for the specific application. In this article, attempts have been made to demonstrate the possible number of FLCs for the given choice. It is institutively felt that as compared to the selection of the fuzzy operators (T-norm, T-conorm, inference engine/fuzzy reasoning), elements of fuzzification (fuzzy sets and their membership functions on inputs and outputs), and defuzzification, it is the type of input and output variables and, corresponding rule mapping significantly influences the right design of the FLCs. Several of the authors in the past have designed the FLCs by considering different elements of the fuzzy logic while retaining the same rule of mapping and the input-output variables, especially for a class of PI/PD/PID controllers. Analytical structures of such fuzzy two/three controllers and simulation studies have been demonstrated by considering the different elements of fuzzy controllers while retaining the same rule of mapping. It is felt that such studies stand as a corollary to the set of all such possible FLCs, wherein the same rule of mapping is considered.

It is strongly felt that therefore, further research is required to arrive at the optimal set of input-output fuzzy mapping or formulation of the rule base. The article also attempts to provide insight to the beginners in this area about the structure of the FLCs and interdependency of the elements thereof in arriving at such a large number of controllers.

## References

[1] Mohan B. M. and Patel A. V., "Analytical Structures and analysis of the simplest fuzzy PD controllers", IEEE Transactions on Systems, Man and Cybernetics, vol. 32, No. 2, pp. 239 - 248, April 2002.

[2] Patel A. V. and Mohan B. M., "Analytical Structures and analysis of the simplest fuzzy PI controllers", Automatica, vol. 38, pp. 981 – 993, June 2002.

[3] Patel A. V., "Analytical structures and analysis of fuzzy PD controllers with multi-fuzzy sets having a variable cross – point level", Fuzzy Sets and Systems, vol. 129, No. 3, pp. 311 – 334, July 2002.

[4] Patel A. V., "Analytical Structures fuzzy PI controllers with multifuzzy sets under various T-norms, T-conorms, and inference methods", International Journal of Fuzzy Systems, vol. 1, No. 3, pp. 93-141, September 2003 (Invited paper).

[5] Patel A. V., "Simplest Fuzzy PI Controllers under various Defuzzification Methods", International Journal of Computational Cognition, vol. 3, No. 1, pp. 21-34, March 2005.

[6] Hao Ying, "A general technique for deriving analytical structure of fuzzy controllers using arbitrary trapezoidal input fuzzy sets and Zadeh AND operator", Automatica, Volume 39, Issue 7, July 2003, Pages 1171-1184.

[7] B. M. Mohan, Arpita Sinha, "Analytical structure and stability analysis of a fuzzy PID controller", Applied Soft Computing, Volume 8, Issue 1, January 2008, Pages 749-758.

[8] Jerry M. Mendel, Hani Hagras, Woei-Wan Tan, William W. Melek, Hao Ying, "Analytical Structure of Various Interval Type-2 Fuzzy PI and Controllers", Chapter 4, onlinelibrary.wiley.com, https://doi.org/10.1002/9781118886540.ch4, 27 June 2014.

[9] Ritu Raj, B. M. Mohan, "Modelling and Analysis of the Simplest Fuzzy PI/PD Controllers of Takagi-Sugeno Type", IFCA Papers online 49-1 (2016), pp. 537-542.

[10] N K Arun, B. M. Mohan, Ritu Raj, "A Nonlinear Fuzzy PID Controller via Algebraic Product AND-Maximum OR-Height Defuzzification", https://www.researchgate.net/publication/314668494, September 2016, DOI: 10.1109/TechSym.2016.7872657.

[11] Chun-Tang Chao, Nana Sutarna, Juing-Shian Chiou, and Chi-Jo Wang, "Equivalence between Fuzzy PID Controllers and Conventional PID Controllers", International Journal of Applied Sciences, 2017, 7, 513; www.mdpi.com/journal/applsci, doi:10.3390/app7060513.