[Manipal Journal of Science and Technology](https://impressions.manipal.edu/mjst)

Volume 5 | Issue 1 Article 5

6-30-2020

Performance Analysis of Coupled Tank System using I-PD Controller

D Sivakumar
Annamalai University, dsk2k5@gmail.com

Recommended Citation

Sivakumar, D (2020) "Performance Analysis of Coupled Tank System using I-PD Controller," *Manipal Journal of Science and Technology*: Vol. 5: Iss. 1, Article 5.
Available at: https://impressions.manipal.edu/mjst/vol5/iss1/5

Research Article

Performance Analysis of Coupled Tank System using I-PD Controller

C Ravikumar, D Sivakumar*

Email: dsk2k5@gmail.com

Abstract

Consider a coupled tank system at its level to be controlled which is an example of an uncertain system and it is also a Multiple-Input Multiple-Output (MIMO) system. This paper is to investigate the performance of the I-PD controller used to control the level of the coupled tank system. The variable to be controlled is the liquid level in tank2 and the manipulated variable is an inflow of tank1. Therefore, any control algorithm developed for the process should consider the interaction between loops. A control mechanism is required which provides a good deal between interaction and performance even in the presence of uncertainty due to model mismatches or environmental conditions specifically, when moderate interaction exists among the process variables. The characteristics of an I-PD controller compared with a PID controller designed based on the Small-gain theorem that will guarantee the closed-loop stability. The performance of the controllers is evaluated using various parameters such as ISE, settling time and percentage overshoot and simulation is carried out with the help of MATLAB - Simulink environment.

Keywords: I-PD controller, Small-gain theorem, Coupled Tank system, closed-loop stability, uncertain system.

Introduction

In this paper, a coupled tank system is used for liquid level control. In process industries such as petroleum refinery, paper and cement industry, water treatment plants and food processing industries, the most common control problems is the liquid level in tanks and flow between tanks. A common tank system required the controller to maintain the level of the liquid at the desired level. A number of conventional controllers are being used in a process industry that exploits several tuning methods for obtaining appropriate control parameters.

The main objective of this paper is to determine the mathematical model of a coupled tank system and design a controller. Here comparative analysis of the transient response obtained by different controllers-Robust PID controller using small-gain theorem and I-PD Controller has been done using MATLAB Simulink software.

Modelling of the processes is very difficult but it is the basic requirement in the design of the controller. Difficulties in modelling are due to a more complex system, poorly understood phenomena, effects of reduced model order, ignorance of certain factors like non-linearity and time delay, unknown disturbance, and noise input.

All these factors indicate the necessity of having a controller which should work well even if the actual system deviates from its original system. And also, it should give a stable closed-loop system response. The controller is designed by considering uncertainty in a process and obtain guaranteed closed-loop stability and such controllers are known as the robust controller.

Robust controllers can be designed using various approaches. A robust control problem is about analysing accurate control systems with

---

C Ravikumar[1], D Sivakumar[2]

[1] *Department of Instrumentation and Control Engineering, A.V.C. College of Engineering.*

[2] *Department of Electronics and Instrumentation Engineering, Annamalai University.*

Manuscript received: 10 April 2020
Revision accepted: 28 April 2020

* Corresponding Author

How to cite this article: C Ravikumar, D Sivakumar. "Performance Analysis of Coupled Tank System using I-PD Controller", Manipal J. Sci. Tech., vol.5(1), 26-31, 2020.

Published by Impressions@MAHE, 2020

uncertainties. Robust controller design can be done using Edge Theorem [2]. The optimal control approach is used to design robust controllers [3].

Dubravska M. and Harsanyi L. [4, 5] have proposed the design of a robust controller for the uncertain system. Kharitonov theorem can also be used to design a robust controller for an uncertain plant [6].

The main downside of the PID controller is the setpoint kick-off. This will damage the final control element like the control valve or motor. To eliminate such a disadvantage, the derivative action is introduced in the feedback path in the I-PD controller which will improve the system response. The I-PD control action can be used in any physical system that uses closed-loop control of output variables such as temperature, level, flow, pressure, speed, position control systems, etc.

Many researchers [7-13] proposed I-PD tuning rules for various stable and non-linear processes. The paper is organized as follows. The modelling of the process is described in detail following the general methodology. Then the design of controllers is presented for the two-tank process followed by the results and conclusions.

Methodology

A. Small-Gain Theorem

One form of uncertainty in dynamical systems is parametric uncertainty in which the parameters describing the system are unknown. A SISO uncertain system G(s) is described by a family of transfer functions and is controlled by a controller C(s) as shown in Fig. (1). The loop transfer function is defined as the product of the forward transfer function and feedback transfer function.

That is $L(s) = C(s)\,G(s)$ (1)

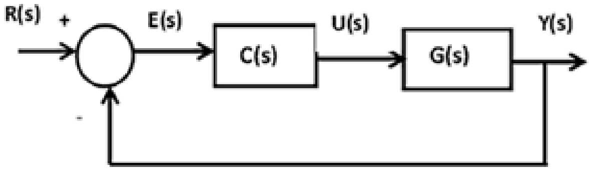

Fig 1 : Closed-loop control system

The Small-Gain theorem states that if the absolute value of the product of the plant and the controller is less than unity. Hence for closed-loop robust stability,

$|L(j\omega)| < 1$ for all $\omega \,\varepsilon\, [0, \infty]$. (2)

If the plant to be controlled is uncertain and described by a number of stable transfer functions be $G_0(s)$, then the perturbed system can be represented in the form of an unstructured additive uncertainty as shown in Fig (2). $W_a(s)$ represents the weighting transfer function and $\Delta(s)$ represents a set of transfer functions with peak magnitudes less than or equal to one for all frequencies.

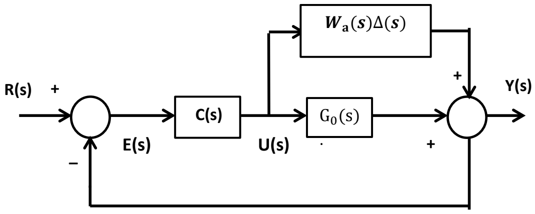

Fig 2 : Closed-loop uncertain system

Following the design procedure presented in [1] the controller parameters can be determined as below,

$k_d = n_d / K,\ K_p = n_p/K,\ K_i = n_i/K$ (3)

B. I-PD Controller

It is the modified structure of the PID controller that is mentioned as a brilliant PID controller which reduces peak overshoot. In the I-PD controller, the integral term is acting on the error e(t) and the proportional plus derivative term is acting on the process variable y(t).

The output of the controller is given by

$u(t) = K_pK_p\ [T_iT_i \int e(t)dt - (y(t) + T_dT_d \frac{dy(t)dy(t)}{dt\quad dt})]$ (4)

Where,

Kp - Proportional gain

Ti - Integral time

Td - Derivative time

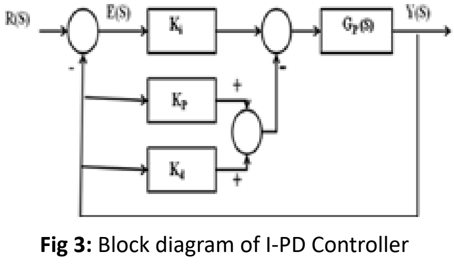

Fig 3: Block diagram of I-PD Controller

Description Of The Process

The liquid tank process plays a vital role in industrial applications such as food processing, filtration, industrial chemical processing, spray coating, and pharmaceutical industries. Many industrial applications are concerned with liquid level control. It may be a single loop level control or sometimes multi-loop level control.

In this work, the coupled tank is considered as a SISO system with h_2 as the process variable and q_{in1} as the manipulated variable. The experimental setup consists of two tanks as shown in Fig (4). The two tanks can be coupled using valve q_{12} is a manual valve. Therefore, the coupled tank process with two tanks represents a Multi-Input Multi-Output (MIMO) system for opened valve q_{12} or two independent Single-Input Single-Output (SISO) systems for closed valve q_{12}.

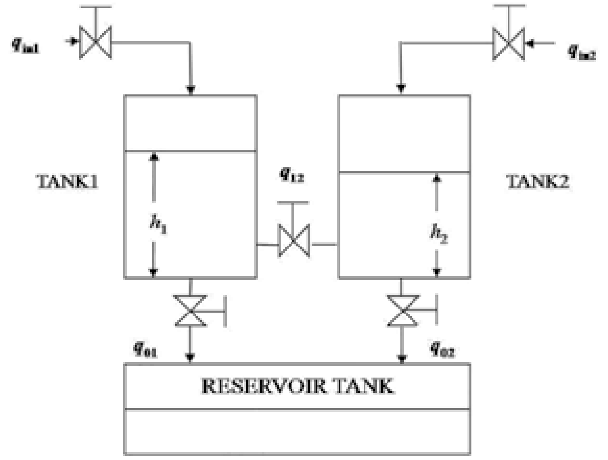

Fig 4: Coupled Tank Process

The mass balance equation of the tank process is given by,

$$\frac{dh_1}{dt} = \frac{q_{in1}}{A_1} - \frac{a_1}{A_1}\sqrt{2gh_1} - \frac{a_{12}}{A_1}\sqrt{2g[h_1-h_2]} \quad (5)$$

$$\frac{dh_2}{dt} = \frac{q_{in2}}{A_2} - \frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_{12}}{A_2}\sqrt{2g[h_1-h_2]} \quad (6)$$

Where,

q_{in1} - Inflow to the tank1

q_{in} - Inflow to the tank2

h_1 - Height of tank1

h_2 - Height of tank2

A_1 - Area of tank1

A_2 - Area of tank2

a_1 - Area of the pipe outlet1

a_2 - Area of the pipe outlet2

a_{12} - Area of the pipe connecting Tank-1 and Tank-2

g - Acceleration due to gravity

The white box model is developed using SIMULINK software with the following specifications:

A_1 and A_2 = 1130.4 cm^2
h_1 and h_2 = 25 cm
a_1 and a_2 = 7.8 cm^2
a_{12} = 1.274 cm^2
Maximum q_{in1} = 100 LPH

The operating condition of the process is fixed as,
q_{in1} = 50 LPH
h_1 = 11.85 cm

With the area of the outlets of tank1 and tank2 as
a_1 = 5 cm^2
a_2 = 0.5 cm^2 and
a_{12} = 1.274 cm^2

The studies are carried out using three different operating conditions for the chosen process so as to understand the complete characteristics of the process and also to test the stability conditions.

For performance analysis the following open-loop responses are obtained in this work, i) Nominal model of the process is considered under the normal operating conditions with 50% input, ii) open-loop response with 10% change in input in both the directions from the nominal value.

The nominal model of the process is obtained using a step test signal under the nominal operating conditions as shown in Fig (5).

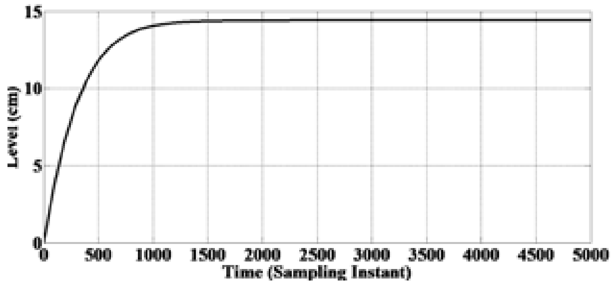

Fig 5: Open-loop response of the process with 50% input

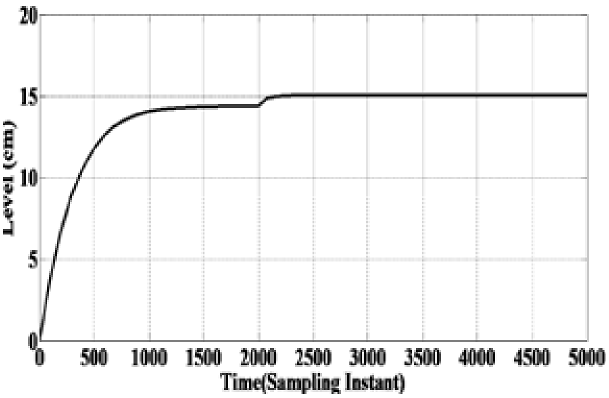

Fig 6: Open-loop response of the process with 10% change in input from Nominal operating condition

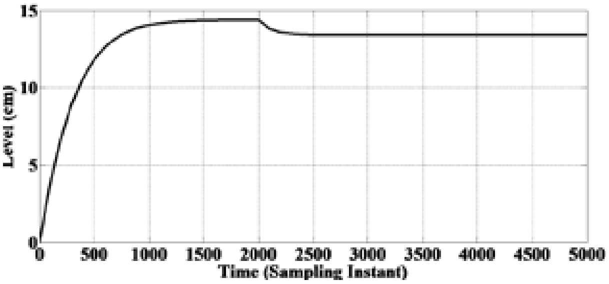

Fig 7: Open-loop response of the process with -10% change in input from Nominal operating condition

Corresponding transfer functions are,

$G_0 G_0(s) = \frac{28.8}{29.6s^2+14.7s+1}$ (Nominal Model)

$G_1 G_1(s) = \frac{6.5}{609.5s^2+69.7s+1}$ (10% change in input from operating condition)

$G_2 G_2(s) = \frac{10}{627.13s^2+113.75s+1}$ (-10% change in input from operating condition)

Controller Design

A. Robust PID Controller

From figure (2),

$$G(s) = G_0(s) + Wa(s).\ \Delta(s) \qquad (7)$$

$$\text{if } |\Delta(s)| \leq 1 \text{ then, } |Wa(s).\ \Delta(s)| \leq |Wa(s)| \qquad (8)$$

The weighting transfer function is chosen such that the following condition is satisfied.

$$|Wa(s)| \geq \text{Max } |G_k(s)-G_0(s)| \text{ for all } \omega \in [0, \infty] \qquad (9)$$

where k = 1, 2, 3, ... n
$G_k(s)$ represents the set of n stable transfer functions describing the uncertain system.
The nominal model of the coupled tank process is given by,

$$G_0 G_0(s) = G_{0c} G_{0c}(s)/\ G_{0m}\ G_{0m}(s) \qquad (10)$$

Let the PID controller to be designed be given by,

$$C(s) = C_c C_c(s)/\ C_m C_m(s) = (n_d s2 + n_p s + n_i)/Ks \qquad (11)$$

Using the design procedure, the PID controller parameters are determined and given below,

$$n_p n_p = 88.20;\ n_i n_i = 1;\ n_d n_d = 1 \qquad (12)$$

K = 1 and suitable Wa is selected so that the stability condition said in [1]
$G_{NCL}(s)| < |G_0(s)/W_a(s)|$ is satisfied.

B. I-PD Controller

I-PD controller parameters for each condition are obtained using Ziegler Nichol's tuning method. Table 1 shows the controller parameter values of various changes in input.

Table 1: Tuning parameters of the I-PD controller.

Condition	K_p	T_i	T_d
Nominal Condition	6.94	4.44	1.11
10% change in input	7.53	19.4	4.85
-10% change in input	22.02	11.8	2.95

Results

The coupled tank process is represented as a white box model and executed using SIMULINK software. The process is maintained at the nominal operating condition and the servo and regulatory responses are obtained using robust PID and I-PD controllers for a change in setpoint of 2 cm in both directions. These responses are presented in Fig. (8-10).

The response for setpoint tracking in both directions is presented in Fig. (11 to 13). The performance of the process with two different controllers is evaluated using Settling time, ISE, Rise time.

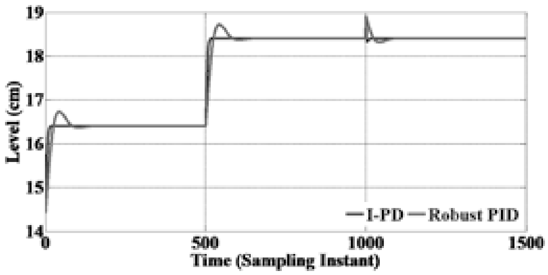

Fig 8: Servo and Regulatory responses of the process with I-PD and robust PID controllers (Under nominal operating conditions).

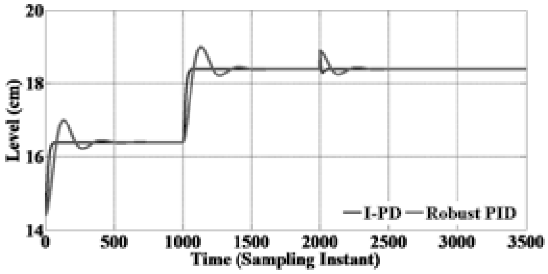

Fig 9: Servo and Regulatory responses of the process with I-PD and robust PID controllers (Model with 10% change in input).

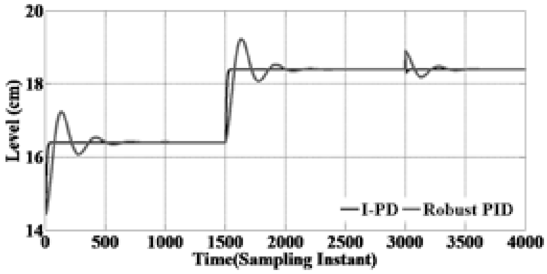

Fig 10: Servo and regulatory responses of the process with I-PD and robust PID controllers (Model with -10% change in input).

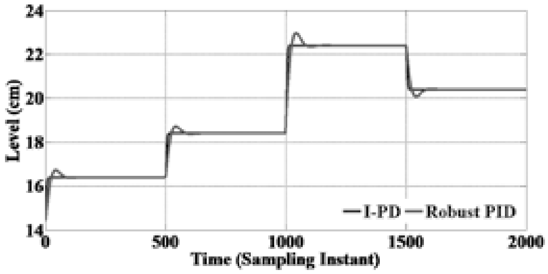

Fig 11: Set Point Tracking responses with I-PD and robust PID controllers (under nominal operating conditions).

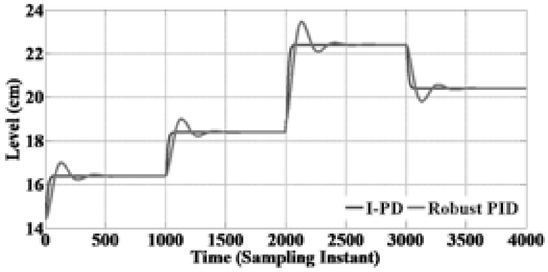

Fig 12: Set Point Tracking responses with I-PD and robust PID controllers (Model with 10% change in input).

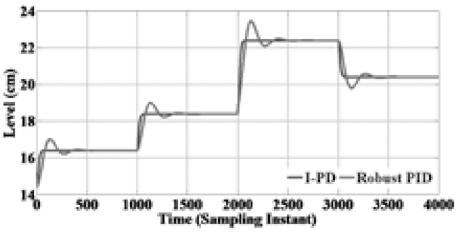

Fig 13: Set Point Tracking responses with I-PD and robust PID controllers (Model with -10% change in input).

Table 2: Performance Evaluation of the process with designed controllers.

Condition	I-PD Controller			Robust PID Controller		
	Settling Time	Rise Time	ISE	Settling Time	Rise Time	ISE
Nominal Operating Condition	30	24.5	11.46	260	208	33.07
10% change in input	100	95	54.29	800	635	171.1
-10% change in input	60	55	31.64	1100	800	196.7

Conclusion

The I-PD Controller and Robust PID controller using Small-Gain Theorem is designed and implemented over coupled tank process. I-PD controller performance is compared with the Robust PID controller. The simulation is carried out in a MATLAB SIMULINK environment. The simulation results are presented along with the performance evaluation shown in Table 2. It is observed from the results that the performance of the process with the I-PD controller is better than the robust PID controller in terms of settling time, in view of peak overshoot, settling time, Rise time, and ISE. In I-PD Controller, the percentage overshoot is eliminated and less settling time when compared with a robust PID controller.

References

[1] Supanna S. Kumar, C. Shreesha, "Robust Controller Design for Uncertain Systems," International journal of scientific engineering and technology, vol. 3, no.2, pp. 207-210, February 2014.

[2] Ivan Holic, Vojtech Vesely, "Robust PID Controller Design for Coupled Tank Process," 18th International Conference on Process Control, June 2011.

[3] Feng Lin, "An Optimal Control Approach to Robust Control Design," INT. J. CONTROL, vol.73, no.3, pp. 177-186.

[4] L. Harsanyi, M. Dubravska, "Robust Controller Design for Linear Systems with Parametric and Dynamic Uncertainties", J. Electrical Engineering, 52 No. 9-10, pp. 307-310, 2001.

[5] Maria Dubravska and Ladislav Harsanyi, "Control of Uncertain systems", Journal of

Electrical Engineering, Vol. 58, No. 4, pp. 228-231, 2007.

[6] Ying J. Huang and Yuan-Jay Wang, "Robust PID tuning strategy for uncertain plants based on the Kharitonov theorem", ISA Transactions 39, pp. 419-431, 2000.

[7] S. Meena, K. Chitra and R. Vijay Anand, "Development of I-PD controller on embedded platform," International conference on signal processing, communication, power, and embedded system, SCOPES-2016.

[8] Sisir Mazumder, Susovan Dutta, "Analytical study and designing of a I-PD Controller (a Practical Modified PID Controller) for a third order system using MATLAB simulation," International Journal of Engineering Research and general science, vol3, Issue 1, Jan-Feb 2015.

[9] Morimasa OGAWA, Tohru KATAYAMA, "A Robust Tuning method for I-PD controller incorporating a constraint on Manipulated Variable," Trans. of the Society of Instrument and Control Engineers, vol. E-1, no.1, 265/273 (2001).

[10] S. J. Suji Prasad, P. A. Balakrishnan, "PSO Based I-PD Controller for Barrel Temperature Control in Plastic Injection Molding System," European Journal of Scientific Research, ISSN 1450-216X, vol.80, no.3 (2012), pp.351-357.

[11] Galal A. Hassaan, "Robustness of I-PD, PD-PI and PI-PD Controllers Used with Second-Order Processes," International Journal of Scientific & Technology Research, ISSN 2277-8616, vol 3, Issue 10, October 2014.

[12] V. Rajinikanth, K. Latha, "I-PD Controller Tuning for Unstable System Using Bacterial Foraging Algorithm: A Study Based on Various Error Criterion," Applied Computational Intelligence and Soft Computing, 2012.

[13] D. Dinesh Kumar, B. Meenakshipriya and S. Sakthiya Ram, "Design of PSO based I-PD Controller and PID Controller for a Spherical Tank System" Indian Journal of Science and Technology, vol 9(12), ISSN: 0974-5645, March 2016.

[14] Kalyan Chakravarthi M, Pannem K Vinay and Nithya Venkatesan, "Design and Simulation of Internal Model Controller for a Real Time Nonlinear Process," Indian Journal of Science and Technology, August 2015.

[15] Aidan O'Dwyer, "Handbook of PI and PID Controller Tuning Rules," 2nd edition.